Funneling of Electromagnetic Energy Through a Rectangular Hole

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When light passes through a subwavelength-scale square hole in a perfect conductor, the flow of electromagnetic energy exhibits a strikingly rich behavior that depends on the geometry of the rectangular hole. We present a detailed calculation and demonstrate that funneling of electromagnetic energy occurs when certain resonance conditions are met.

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I. INTRODUCTION

Strong excitation of a electric field is generally accompanied by a strong build up of electric charge as in the case of a charged capacitor [1]. In the presence of incident electromagnetic waves, electric currents are induced on a metal surface, and charges can be accumulated at the junction blocking the flow of current. Strong build up of charge is expected when the junction has a sharp edge or more importantly when the system is in resonance with the exciting external source. Recently, localized plasmons in nano-scale metallic structures have witnessed an explosion of research efforts with applications in various fields of science, including physics [2–5], chemistry, and biomedical sciences [6–9]. Enhanced transmission of light through holes that are periodically perforated in a metal film demonstrated by Ebbesen, heralded many of these plasmonic efforts [10]. The physical origin of enhanced transmission is the excitation of surface plasmons coupled to incoming and outgoing radiations that are also in resonance with the periodic structures [2, 4, 11–14]. In the case of the Ebbesen structure, the holes are too small to support propagating mode, and light passes through the holes only via a tunneling process. When the size of the hole becomes larger, of the order of the wavelength, propagating modes become available, and a single hole alone can admit resonance. The single-hole resonance and subsequent enhanced transmission is equally important in surface plasmon applications.

In this paper, we consider a single rectangular hole and provide a detailed analysis of the field enhancement. We show that resonance occurs when the long side of a

rectangle is about half the wavelength. The dependence of the enhancement on the size of the rectangle and on the thickness of metal film is obtained, and the nature of the resonance is clarified.

II. FIELD ENHANCEMENT

Consider a metal film with a rectangular hole of size $a \times b$, as shown in Fig. 1, and TM-polarized light incident on it. Field configurations can be obtained by solving Maxwell's equation with appropriate boundary conditions imposed. For simplicity, we assume that metal to be a perfect conductor and impose a vanishing boundary condition for the tangential component of the electric field. Note that TM-polarized light incident obliquely on a metal film with rectangular holes couples to the TE modes of the rectangular holes. Inside the hole ($-h/2 < z < h/2$, which we call region II), we make a single-mode approximation and take into account only the lowest TE_{01} mode inside a rectangular hole of size

Fig. 1. A metal film with a rectangular hole of size $a \times b$.

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Fig. 2. (Color online) Cross-sectional distribution of the electric field near a rectangular hole of $a = \lambda/2$, $b = \lambda/60$, and $h = \lambda/30$, calculated by using a finite-difference time-domain (FDTD) method, for light incident with the resonant wavelength λ .

$a \times b$ ($a > b$), as given by

$$\begin{aligned}
 E_x^{II} &= 0, \\
 E_y^{II} &= \sin\left(\frac{\pi x}{a}\right) \left[-E_L \frac{\cos(\mu z)}{\sin(\mu h/2)} + E_R \frac{\sin(\mu z)}{\cos(\mu h/2)} \right], \\
 E_z^{II} &= 0, \\
 H_x^{II} &= \frac{i\mu}{k_0} \sqrt{\frac{\epsilon_0}{\mu_0}} \sin\left(\frac{\pi x}{a}\right) \\
 &\quad \times \left[E_L \frac{\sin(\mu z)}{\sin(\mu h/2)} + E_R \frac{\cos(\mu z)}{\cos(\mu h/2)} \right], \\
 H_y^{II} &= 0, \\
 H_z^{II} &= -\frac{i\pi}{k_0 a} \sqrt{\frac{\epsilon_0}{\mu_0}} \cos\left(\frac{\pi x}{a}\right) \\
 &\quad \times \left[-E_L \frac{\cos(\mu z)}{\sin(\mu h/2)} + E_R \frac{\sin(\mu z)}{\cos(\mu h/2)} \right], \quad (1)
 \end{aligned}$$

where $\mu = \sqrt{k_0^2 - (\pi/a)^2}$.

The incoming TM-polarized light and the reflected light in the region $z > h/2$ (which we call region I in Fig. 2) can be expressed as follows:

$$\begin{aligned}
 H_x^I &= \sqrt{\frac{\epsilon_0}{\mu_0}} \int dk_x dk_y \left[g_x(k_x, k_y) e^{i\Theta + ik_z(z-h/2)} \right. \\
 &\quad \left. + \delta(k_x) \delta(k_y) e^{-ik_z(z-h/2)} \right], \\
 H_y^I &= \sqrt{\frac{\epsilon_0}{\mu_0}} \int dk_x dk_y \left[g_y(k_x, k_y) e^{i\Theta + ik_z(z-h/2)} \right], \\
 H_z^I &= \sqrt{\frac{\epsilon_0}{\mu_0}} \int dk_x dk_y \left[g_z(k_x, k_y) e^{i\Theta + ik_z(z-h/2)} \right], \quad (2)
 \end{aligned}$$

where

$$\begin{aligned}
 \Theta &= k_x(x - a/2) + k_y(y - b/2), \\
 k_z^2 &= k_0^2 - k_x^2 - k_y^2. \quad (3)
 \end{aligned}$$

The sign of k_z is chosen such that the imaginary part of k_z is positive. In the same manner, we take the expressions for the transmitted light in the region $z < -h/2$

(region III) to be

$$\begin{aligned}
 H_x^{III} &= \sqrt{\frac{\epsilon_0}{\mu_0}} \int dk_x dk_y \left[f_x(k_x, k_y) e^{i\Theta - ik_z(z+h/2)} \right], \\
 H_y^{III} &= \sqrt{\frac{\epsilon_0}{\mu_0}} \int dk_x dk_y \left[f_y(k_x, k_y) e^{i\Theta - ik_z(z+h/2)} \right], \\
 H_z^{III} &= \sqrt{\frac{\epsilon_0}{\mu_0}} \int dk_x dk_y \left[f_z(k_x, k_y) e^{i\Theta - ik_z(z+h/2)} \right]. \quad (4)
 \end{aligned}$$

With field configurations in each region satisfying Maxwell's equation separately, the overall solution is obtained by matching the boundary conditions at the metal surface and at the interface $z = \pm h/2$. The vanishing tangential electric field on a metal surface requires

$$\begin{aligned}
 \nabla \times \vec{H}|_{\text{metal}} &= 0 \rightarrow \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right)|_{\text{metal}} = 0, \\
 \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right)|_{\text{metal}} &= 0 \quad (5)
 \end{aligned}$$

so that

$$\begin{aligned}
 \int dk_x dk_y (ik_y f_z + ik_z f_y) e^{i\Theta} &= 0, \\
 \int dk_x dk_y (ik_y g_z - ik_z g_y) e^{i\Theta} &= 0. \quad (6)
 \end{aligned}$$

The y -component of the electric field vanishes on a metal surface and matches with the internal mode at the interface such that

$$\begin{aligned}
 &\int dk_x dk_y (ik_z f_x + ik_x f_z) e^{i\Theta} \\
 &= -ik_0 \sin\left(\frac{\pi x}{a}\right) [E_L \cot(\mu h/2) + E_R \tan(\mu h/2)] \\
 &= 0 \text{ otherwise,} \\
 &\int dk_x dk_y [ik_z g_x - ik_0 \delta(k_x) \delta(k_y) - ik_x g_z] e^{i\Theta} \\
 &= ik_0 \sin\left(\frac{\pi x}{a}\right) [E_L \cot(\mu h/2) - E_R \tan(\mu h/2)] \\
 &= 0 \text{ otherwise.} \quad (7)
 \end{aligned}$$

The divergencelessness of $\vec{H}$ ($\nabla \cdot \vec{H} = 0$) gives rise to

$$\begin{aligned}
 f_z &= \frac{1}{k_z} (k_x f_x + k_y f_y) = \frac{k_x k_z}{k_y^2 + k_z^2} f_x, \\
 f_y &= -\frac{k_x k_y}{k_y^2 + k_z^2} f_x, \\
 g_z &= -\frac{1}{k_z} (k_x g_x + k_y g_y) = -\frac{k_x k_z g_x}{k_z^2 + k_y^2}, \\
 g_y &= -\frac{k_x k_y g_x}{k_z^2 + k_y^2}. \quad (8)
 \end{aligned}$$

On the other hand, the continuity of $\vec{H}$ at the interface requires that

$$\begin{aligned}
 \frac{i\mu}{k_0} \sin\left(\frac{\pi x}{a}\right) [-E_L + E_R] &= \int dk_x dk_y f_x e^{i\Theta}, \\
 \frac{i\mu}{k_0} \sin\left(\frac{\pi x}{a}\right) [E_L + E_R] &= \int dk_x dk_y g_x e^{i\Theta} + 1, \quad (9)
 \end{aligned}$$

which, integrating over x after multiplying by the factor $\sin(\pi x/a)$, can be converted to

$$\begin{aligned} \frac{i\mu}{k_0} \frac{ab}{2} [-E_L + E_R] &= \int dk_x dk_y f_x \frac{ab}{2} J, \\ \frac{i\mu}{k_0} \frac{ab}{2} [E_L + E_R] &= \int dk_x dk_y g_x \frac{ab}{2} J + \frac{2ab}{\pi}, \\ J &= \frac{2}{ab} \int_0^a dx \int_0^b dy \sin\left(\frac{\pi x}{a}\right) e^{i\Theta}. \end{aligned} \quad (10)$$

Similarly, by taking the inverse Fourier transform of Eq. (7), we convert f_x and g_x :

$$\begin{aligned} f_x &= -\frac{k_y^2 + k_z^2}{k_0 k_z} K [E_L \cot(\mu h/2) + E_R \tan(\mu h/2)], \\ g_x &= \frac{k_y^2 + k_z^2}{k_0 k_z} [\delta(k_x) \delta(k_y) \\ &\quad + K (E_L \cot(\mu h/2) - E_R \tan(\mu h/2))], \\ K &= \frac{1}{4\pi^2} \int_0^a dx \int_0^b dy \sin\left(\frac{\pi x}{a}\right) e^{-i\Theta}. \end{aligned} \quad (11)$$

Combing them with Eq. (10), we finally solve for E_L and E_R :

$$\begin{aligned} E_L &= \frac{1}{D} [\mu \sin(\mu h) - 2ik_0 W \sin^2(\mu h/2)], \\ E_R &= \frac{1}{D} [\mu \sin(\mu h) + 2ik_0 W \cos^2(\mu h/2)], \end{aligned} \quad (12)$$

where

$$\begin{aligned} D &= \frac{i\pi}{4k_0} [\sin(\mu h)(\mu^2 + k_0^2 W^2) + 2i\mu k_0 W \cos(\mu h)], \\ W &= \int dk_x dk_y \frac{k_y^2 + k_z^2}{k_0 k_z} \frac{ab}{8\pi^2} J^2 \approx \frac{32ab}{3\pi} k_0^2, \\ J &= \frac{2}{ab} \int_0^a dx \int_0^b dy \sin\left(\frac{\pi x}{a}\right) e^{i\Theta} \\ &= \text{sinc}(bk_y/2) [\text{sinc}(\pi/2 + ak_x/2) \\ &\quad + \text{sinc}(\pi/2 - ak_x/2)]. \end{aligned}$$

III. FUNNELING OF THE ELECTROMAGNETIC FIELD

Having determined all relevant coefficients, the solution of Maxwell's equation for TM light incident on a rectangular hole is uniquely found up to an overall constant characterizing the strength of the incoming wave. In particular, the electromagnetic field inside the rectangular hole is given by

$$\begin{aligned} E_y &= -\frac{8k_0}{i\mu\pi} \sin\left(\frac{\pi x}{a}\right) \\ &\quad \times \frac{i\mu \cos(\mu z + \mu h/2) + k_0 W \sin(\mu z + \mu h/2)}{i\mu(1 + \frac{k_0^2 W^2}{\mu^2}) \sin(\mu h) - 2k_0 W \cos(\mu h)}, \end{aligned}$$

Fig. 3. Result from a FDTD calculation showing the Poynting vector, S_z , integrated over the area of the hole at the resonant wavelength. For a simple case, a and h are set to 100 μm and 1 μm , respectively.

$$\begin{aligned} H_x &= \frac{8}{\pi} \sqrt{\frac{\epsilon_0}{\mu_0}} \sin\left(\frac{\pi x}{a}\right) \\ &\quad \times \frac{i\mu \sin(\mu z + \mu h/2) - k_0 W \cos(\mu z + \mu h/2)}{i\mu(1 + \frac{k_0^2 W^2}{\mu^2}) \sin(\mu h) - 2k_0 W \cos(\mu h)}. \end{aligned} \quad (13)$$

These fields determine the electromagnetic energy flow in terms of the Poynting vector

$$\begin{aligned} S_z &= \frac{1}{2} \text{Re}(E_x H_y^* - E_y H_x^*) = -\frac{1}{2} \text{Re}(E_y H_x^*) \\ &= \frac{1}{2} \text{Re} \left[\frac{i\mu}{k_0} \sqrt{\frac{\epsilon_0}{\mu_0}} \sin^2\left(\frac{\pi x}{a}\right) \frac{E_L^* E_R - E_L E_R^*}{\sin(\mu h)} \right] \\ &= -\frac{16}{\pi^2} \frac{\mu^2 \sqrt{\frac{\epsilon_0}{\mu_0}} \sin^2\left(\frac{\pi x}{a}\right) k_0^2 (W + W^*)}{|\sin(\mu h)(\mu^2 + k_0^2 W^2) + 2i\mu k_0 W \cos(\mu h)|^2}. \end{aligned} \quad (14)$$

This is the main result of the present paper. Note that the Poynting vector S_z is independent of the longitudinal direction z , which is consistent with constant energy flow along the longitudinal direction. When μ is small ($\mu \ll k_0$), the total energy flow is

$$\begin{aligned} E_{tot} &= \int_0^a dx \int_0^b dy S_z \\ &\approx -\frac{8}{\pi^2} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\mu^2 ab k_0^2 (W + W^*)}{|\mu h(\mu^2 + k_0^2 W^2) + 2i\mu k_0 W|^2} \\ &\approx -\frac{512}{3\pi^3} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{a^2 b^2 k_0^4}{h^2 [k_0^2 - \pi^2/a^2]^2 + [64abk_0^3/3\pi]^2}. \end{aligned} \quad (15)$$

This shows that the transmitted energy reaches maximum near the half wavelength resonance $k_0 \approx \pi/a$. Near the resonance,

$$E_{tot} \approx -\frac{512}{3\pi^3} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{a^2 b^2 k_0^4}{[64abk_0^3/3\pi]^2} = -\frac{3}{8\pi} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{1}{k_0^2}. \quad (16)$$

This shows a striking result that the total energy remains constant irrespective of the size of the area ab . That is,

Fig. 4. FWHM of the spectrum of total energy flow from a hole with $a = 100 \mu\text{m}$ and $h = 20 \mu\text{m}$.

the total energy does not change, not even with decreasing width b . This fact is consistent with the results from the FDTD calculation, as shown in Fig. 3. Thus, electromagnetic energy, indeed, funnels through a rectangular hole near resonance.

Finally, we point out that the spectrum of the total energy flow is narrower with small a , as Eq. (16) and (17) predict. Shown in Fig. 4 is the full width at half maximum (FWHM) of the spectrum of the total energy flow from a hole, which shows that a small b yields a narrower spectrum.

IV. CONCLUSION

In conclusion, we have shown that resonant funneling of an electromagnetic wave occurs when the wavelength is about twice as long as the long side of a rectangular hole. The energy transmitted through the rectangular hole at the resonant wavelength remains constant and is independent of the size of the hole.

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