Phases of the Brans-Dicke Cosmology with Matter

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We study the cosmology of the Brans-Dicke theory with perfect fluid-type matter. In our previous work, we found exact solutions for any Brans-Dicke parameter ω and for the general parameter γ of the equation of state. In this paper, we further study the cosmology of those solutions by analyzing them according to their asymptotic behaviors. The cosmology is classified into nineteen phases according to the values of γ and ω . The effect of the cosmological constant on the Brans-Dicke theory is a particular case of our model. We plot the time evolution of the scale factor by using numerical investigations. We also compare the solutions for the theories with and without matter.

I. INTRODUCTION

Recent developments of the string theory suggest that in the region of Planck length curvature, the quantum fluctuation is very large so that string coupling becomes large and, consequently, the fundamental string degrees of freedom are not weakly coupled *good* ones [1]. Instead, solitonic degrees of freedom, like p-brane or D-p-brane [2], are more important. Therefore, it is a very interesting question to ask what is the effect of these new degrees of freedom on the space-time structure, especially whether including these degrees of freedom resolves the initial singularity, which is a problem in standard general relativity. A new gravity theory that can deal with such a new degree of freedom should be a deformation of the standard general relativity so that in a certain limit it should reduce to the standard Einstein theory. The Brans-Dicke theory [3] is a generic deformation of general relativity and allows variable gravity coupling. Therefore, whatever the motivation for modifying the Einstein theory, the Brans-Dicke theory is the first one to be considered. As an example, the low-energy limit of the string theory contains the Brans-Dicke theory with a fine-tuned deformation parameter ($\omega = -1$).

Without knowing the exact theory of the p-brane cosmology, the best guess is that it should be a Brans-Dicke theory with matter. In fact, there is some evidence for this [4]: it is found that the natural metric that couples to the p-brane is the Einstein metric multiplied by a certain power of the dilaton field. In terms of this new metric, the action that gives the p-brane solution becomes the Brans-Dicke action with a definite deformation parameter ω depending on p. Using this action, Rama [5] recently argued that the gas of the solitonic p-brane [4] treated as perfect-fluid type matter in a Brans-Dicke theory can resolve the initial singularity without any explicit solution. In previous papers [6,7], we studied that model and found exact cosmological solutions for any Brans-Dicke parameter ω and for a general equation of state and we classified the cosmology of the solutions according to the range of the parameters involved.

In this paper, we further study the cosmology of those solutions by analyzing them according to their asymptotic behaviors. The cosmology is classified into nineteen phases according to the values of γ and ω . The effect of the cosmological constant on the Brans-Dicke theory is a particular case of our model. We plot the time evolution of the scale factor by using numerical investigations. We also compare the solutions for the theories with and without matter. See also Ref. 8.

The rest of this paper is organized as follows. In Section II, we set up the notation and review our previous results [6,7]. In Section III, we describe two new phases which were not mentioned in Ref. 6, and we classify the cosmology into nineteen phases. Using numerical and analytical methods, we present the behaviors of the scale factor in figures. In Section IV, we present a summary and then conclude with some discussion.

II. BRANS-DICKE COSMOLOGY WITH MATTER

First, we briefly review our earlier work [6]. We consider the Brans-Dicke theory and analyze the evolution of the D-dimensional homogeneous isotropic universe with perfect fluid-type matter. The action is given by

$$S = \int d^D x \sqrt{-g} e^{-\phi} \left[\mathcal{R} - \omega \partial_\mu \phi \partial^\mu \phi \right] + S_m, \tag{1}$$

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Journal of the Korean Physical Society, Vol. 34, No. 5, May 1999

-0.5 \mathbf{x} 0 0.5 \mathbf{y} \mathbf{y} \mathbf{x} \mathbf{y} \mathbf{x} \mathbf{y} \mathbf{y} \mathbf{x} \mathbf{y} \mathbf{y} \mathbf{x} \mathbf{y} \mathbf{y}

Fig. 1. Phase diagram of 11 regions.

where ϕ is the dilaton field and S_m is the matter part of the action. Here, we assume that the matter has no dilaton coupling.

Let's assume that the matter can be treated as a perfect fluid with the equation of state

$$p = \gamma \rho, \quad \gamma < 1. \tag{2}$$

Therefore, our starting point is the equation of the Brans-Dicke theory [9,10]

$$\mathcal{R}_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{R} = \frac{e^{\phi}}{2} T_{\mu\nu} + \omega \left\{ \partial_{\mu} \phi \partial_{\nu} \phi - \frac{g_{\mu\nu}}{2} (\partial \phi)^2 \right\} \\ + \left\{ -\partial_{\mu} \partial_{\nu} \phi + \partial_{\mu} \phi \partial_{\nu} \phi + g_{\mu\nu} \mathcal{D}^2 \phi - g_{mu\nu} (\partial \phi)^2 \right\}, \\ 0 = \mathcal{R} - 2\omega \mathcal{D}^2 \phi + \omega (\partial \phi)^2, \tag{3}$$

where ϕ is the dilaton and \mathcal{D} means a covariant derivative. \mathcal{R} is the curvature scalar, and the metric is given as

$$ds^{2} = -\frac{1}{N}dt^{2} + E^{2\alpha(t)}\delta_{ij}dx^{i}dx^{j}$$
(4)
(*i*, *j* = 1, 2, ..., *D* - 1),

where $e^{\alpha(t)}(=a(t))$ is the scale factor and \mathcal{N} is the (constant) lapse function.

The energy-momentum tensor is given by

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)U_{\mu}U_{\nu},$$
 (5)

where U_{μ} is the fluid velocity. The hydrostatic equilibrium condition of energy-momentum conservation is

$$\dot{o} + (D-1)(p+\rho)\dot{\alpha} = 0.$$
 (6)

Using the equation of state, Eq. (2), with a free parameter γ , we get the solution

$$\rho = \rho_0 e^{-(D-1)(1+\gamma)\alpha},\tag{7}$$

where ρ_0 is a real number. Our goal is to study how the metric variables change their behaviors for various values of γ and ω . Now, since we consider only the time dependence, the action can be brought to the following form:

$$S = \int dt \ e^{(D-1)\alpha-\phi} \left[\frac{1}{\sqrt{\mathcal{N}}} \left\{ -(D-2)(D-1)\dot{\alpha}^2 + 2(D-1)\dot{\alpha}\dot{\phi} + \omega\dot{\phi}^2 \right\} - \sqrt{\mathcal{N}}\rho_0 e^{-(D-1)(1+\gamma)\alpha+\phi} \right], \quad (8)$$

where we have eliminated p and ρ by using Eq. (6). After getting the constraint equation by varying over the constant lapse function, \mathcal{N} , we can set it to be one.

Table. 1. The sign of T_{\pm} determines the range of time t as follows: $t(\tau)$ maps the real line of τ to $(1) (-\infty, \infty)$ if $T_{-} < 0 < T_{+}$, (2) $(-\infty, t_{f}]$ if $T_{-} < 0$ and $T_{+} < 0$, (3) $[t_{i}, \infty)$ if $T_{-} > 0$ and $T_{+} > 0$, and (4) $[t_{i}, t_{f}]$ if $T_{+} < 0 < T_{-}$. The sign of H_{\pm}/T_{\pm} determines the asymptotic behavior of the scale factor a(t).

phase	sign of κ	sign of T_{-}	sign of T_+	range of t	sign of H/T	sign of H_+/T_+
Ι	-	+	-	$[t_i, t_f]$	+	+
II	-	-	-	$(-\infty, t_f]$	-	+
III^{-}	+	+		$[t_i, t_f]$	+	
III^+	+		+	$[t_i,\infty)$		+
IV	-	-	+	$(-\infty,\infty)$	+	+
V	-	+	+	$[t_i,\infty)$	-	+
VI	-	+	+	$[t_i,\infty)$	+	+
VII^{-}	+	+		$[t_i,\infty)$	+	
VII^+	+		+	$(-\infty,\infty)$		+
$VIII^{-}$	+	+		$[t_i,\infty)$	+	
$VIII^+$	+		-	$(-\infty, t_f]$		-
IX^{-}	+	+		$[t_i,\infty)$	+	
IX^+	+		+	$(-\infty,\infty)$		+
X^{-}	+	+		$[t_i,\infty)$	+	
X^+	+		-	$(-\infty, t_f]$		-
XI^-	+	+		$[t_i,\infty)$	+	
XI^+	+		-	$(-\infty, t_f]$		+

Phases of the Brans-Dicke Cosmology with Matter - Chanyong PARK and Sang-Jin SIN



Fig. 2. Phase diagram, 12 regions.

Now, introducing a new time variable τ as

$$d\tau = e^{-(D-1)\alpha + \phi} dt \tag{9}$$

and the new variables

$$X = -\frac{1}{2}[(D-1)(1-\gamma)\alpha - \phi],$$

$$Y = \alpha + \frac{\nu}{\kappa}X,$$
(10)

we can write the action as

$$S = \int d\tau \left[\frac{1}{\sqrt{\mathcal{N}}} \left\{ (D-1)\kappa \dot{Y}^2 + \mu \dot{X}^2 \right\} - \sqrt{\mathcal{N}}\rho_0 e^{-2X} \right], (11)$$

where

$$\kappa = (D-1)(1-\gamma)^2(\omega - \omega_{\kappa}),$$

$$\nu = 2(1 - \gamma)(\omega - \omega_{\nu}),
\mu = -\frac{4(D - 2)}{\kappa}(\omega - \omega_{-1}),
\omega_{\kappa} = -\frac{D - 2D\gamma + 2\gamma}{(D - 1)(1 - \gamma)^{2}},
\omega_{\nu} = -\frac{1}{1 - \gamma},
\omega_{-1} = -\frac{D - 1}{D - 2}.$$
(12)

The constraint equation is given by

$$0 = (D-1)\kappa \dot{Y}^2 + \mu \dot{X}^2 + \rho_0 e^{-2X}.$$
(13)

Note that for D > 2 and $\gamma < 1$, the sign of κ is determined by that of $\omega - \omega_{\kappa}$, and the sign of μ is determined by those of $\omega - \omega_{-1}$ and κ .

The equations of motion are simply

$$0 = \ddot{Y}, 0 = \ddot{X} - \frac{\rho_0}{\mu} e^{-2X}.$$
 (14)

When $\rho_0 = 0$, the situation is that of the string cosmology discussed first in Ref. 11, and the solution for X is $X = c\tau$. One can easily show that this solution has two disconnected branches in terms of the original time t; one is an inflation-type branch and the other is a FRW-type branch. If $\rho_0 \neq 0$, the asymptotic behavior of X is $X \sim c |\tau|$, as we will see later. In other words, the behavior of the cosmology at $\rho_0 \neq 0$ is not continuously connected to that of the cosmology at $\rho_0 = 0$ in $\tau \to -\infty$ region. Some of the new aspects of the cosmology due

Table. 2. All possible phases are classified. Here, phases XII^- and XII^+ are new phases.

H_{+}/T_{+} $0 < H_{+}/T_{+} < 1$
$0 < H_+/T_+ < 1$
$0 < H_+/T_+ < 1$
$0 < H_+/T_+ < 1$
$H_+/T_+ > 1$
$0 < H_+/T_+ < 1$
$0 < H_+/T_+ < 1$
$0 < H_+/T_+ < 1$
$H_+/T_+ > 1$
$H_+/T_+ < 0$
$H_+/T_+ > 1$
$H_+/T_+ < 0$
$0 < H_+/T_+ < 1$
$0 < H_+/T_+ < 1$

-465-



Fig. 3. The behavior of the scale factor from phase I to VI.

to the presence of matter come from this discontinuity. If ρ_0 is a negative constant, then the solution oscillates in time, leading to an unphysical solution. This includes the situation where there is a negative cosmological constant in the Brans-Dicke theory. In this paper, therefore, we will consider only positive ρ_0 .

If ω is less than ω_{-1} , the kinetic term of the dilaton has a negative energy in the Einstein frame. Thus, we will consider the case where ω is larger than ω_{-1} . According to the sign of κ , the types of solutions are very different. When κ is negative, the exact solution is

$$X = \ln \left[\frac{q}{c} \cosh(c\tau) \right],$$

$$Y = A\tau + B,$$
(15)

where c, A, B, and $q = \sqrt{\frac{\rho_0}{|\mu|}}$ are arbitrary real constants. Using the constraint equation, we can determine A in terms of the other parameters:

$$A = \frac{c}{\delta}, \quad \text{with} \quad \delta = \sqrt{-\frac{(D-1)\kappa}{\mu}} = \frac{|\kappa|}{2\sqrt{1+\omega\frac{D-2}{D-1}}}.$$
 (16)

If κ is zero, it turns out that the solution of the equations of motion does not satisfy the constraint equation. If κ is positive, the solution is

$$X = \ln \left[\frac{q}{c} | \sinh(c\tau) | \right],$$

$$Y = \frac{c}{\delta}\tau + B.$$
(17)

III. PHASES OF THE BRANS-DICKE THEORY

In Ref. 6, the behaviors of the scale factor were classified according to ω and γ by using eleven regions in Fig. 1. The asymptotic behaviors of the scale factor $a(\tau)$ and the time $t(\tau)$ as $\tau \to \pm \infty$ were shown to be

$$t - t_0 \approx \frac{1}{T_{\pm}} \left(e^{T_{\pm}\tau} - e^{T_{\pm}\tau_0} \right),$$

$$T_{\pm} = \frac{2c}{|\kappa|} \left[(D-1)\gamma \sqrt{1 + \omega \frac{D-2}{D-1}} \right]$$

$$\mp \operatorname{sign}(\kappa) \{\kappa + (D-1)\gamma(1 + \omega(1-\gamma))\},$$

$$a(\tau) \approx e^{H_{\pm}\tau},$$

$$H_{\pm} = \frac{2c}{|\kappa|} \left[\sqrt{1 + \omega \frac{D-2}{D-1}} \mp \operatorname{sign}(\kappa) \{1 + \omega(1-\gamma)\} \right] (18)$$

Note that the range of t is determined by the sign of T_{\pm} :

 $\begin{array}{ll} (-\infty,\infty) & \text{if } \ T_- < 0 < T_+, \\ (-\infty,t_f) & \text{if } \ T_- < 0 \ \text{and} \ T_+ < 0, \\ (t_i,\infty) & \text{if } \ T_- > 0 \ \text{and} \ T_+ > 0, \\ (t_i,t_f) & \text{if } \ T_+ < 0 < T_-. \end{array}$

For $\kappa > 0$, $t(\tau)$ and $a(\tau)$ behave as [6]

$$t \approx -\text{sign}(\tau) \frac{q^{-\eta} e^{(D-1)\gamma B}}{(\eta-1)} \frac{1}{\mid \tau \mid^{\eta-1}},$$



Fig. 4. The behavior of the scale factor from phase VII^- to X^+ .

$$a \approx e^{B}(q \mid \tau \mid)^{-\frac{2(1-\gamma)(\omega-\omega_{\nu})}{|\kappa|}}, \qquad (19)$$

as τ goes to zero, where $\eta = 2 + \frac{(D-1)\gamma\nu}{\kappa}$. $t(\tau)$ is singular as $\tau \to 0$ if $\eta > 1$. Thus, for $\kappa > 0$ and $\eta > 1$, the scale factor a(t) has two branches. The asymptotic form of a(t) as a function of t is given by

$$a(t) \approx [T_{-}(t-t_i)]^{H_{-}/T_{-}} \quad \text{as} \quad \tau \to -\infty,$$

$$a(t) \approx [T_{+}(t-t_f)]^{H_{+}/T_{+}} \quad \text{as} \quad \tau \to \infty,$$
 (20)

where t_i (t_f) is the starting (ending) point at finite time. Equation (18) contains the cases where t starts from $-\infty$ and/or ends at ∞ by setting $t_i = 0$ and/or $t_f = 0$. According to the signs of T_{\pm} , H_{\pm}/T_{\pm} , and the singularity at $\tau = 0$, we classified the behavior of the Brans-Dicke theory [6].

Here, we summarize the result by Table 1. Now, notice that not only the sign of H_{\pm}/T_{\pm} but also that of $H_{\pm}/T_{\pm}-1$ is important because the universe will accelerate if $H_{\pm}/T_{\pm}-1 > 0$ and decelerate if $H_{\pm}/T_{\pm}-1 < 0$ when $\tau \to \pm \infty$. Therefore, we further classify the phases of cosmology accordingly.



Fig. 5. The behavior of the scale factor from phase XI^- to XII^+ .

-468-

1. Case $\omega < \omega_{\kappa}$

A. $H_{-}/T_{-} > 1$

• For $T_- > 0$, the condition $H_-/T_- > 1$ is reduced to

$$\sqrt{1+\omega\frac{D-2}{D-1}} > \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}.$$
 (21)

If $\gamma < 1/(D-1)$, the condition is automatically satisfied. If $\gamma > 1/(D-1)$, the inequality in Eq. (21) turns out to be reduced to $\omega > \omega_{\kappa}$, which is surprising. This means that there is no solution. Therefore, among the regions I and VI which have $T_{-} > 0$, $H_{-} > 0$, and $\omega < \omega_{\kappa}$, only I satisfies $H_{-}/T_{-} > 1$.

• If $T_{-} < 0$, the condition $H_{-}/T_{-} > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)},$$
(22)

whose solution is $\gamma > 1/(D-1)$ and $\omega < \omega_{\kappa}$. Only IV satisfies the conditions, $\omega < \omega_{\kappa}$, $T_{-} < 0$, $H_{-} < 0$, and $H_{-}/T_{-} > 1$.

B. $H_+/T_+ > 1$

• If $T_+ > 0$, the condition $H_+/T_+ > 1$ implies

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)},$$
(23)

whose solution is given by $\gamma < 1/(D-1)$ and $\omega < \omega_{\kappa}$. There is no region satisfying $T_+ > 0$, $\gamma < 1/(D-1)$, and $\omega < \omega_{\kappa}$.

• If $T_{+} < 0, H_{+}/T_{+} > 1$ is reduced to

$$\sqrt{1+\omega\frac{D-2}{D-1}} > -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)},$$
(24)

whose solution is $\gamma > 1/(D-1)$ or $\omega > \omega_{\kappa}$ for $\gamma < 1/(D-1)$. There is no region satisfying the conditions $\omega < \omega_{\kappa}$, $T_{+} < 0$, and $H_{+}/T_{+} > 1$.

2. Case $\omega > \omega_{\kappa}$

A. $H_{-}/T_{-} > 1$

Journal of the Korean Physical Society, Vol. 34, No. 5, May 1999

• For $T_- > 0$, the condition $H_-/T_- > 1$ is reduced to

$$\sqrt{1+\omega\frac{D-2}{D-1}} < -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)},$$
(25)

whose solution is $\gamma < 1/(D-1)$ and $\omega < \omega_{\kappa}$. Therefore, there is no solution satisfying the conditions: $\omega < \omega_{\kappa}$ and $H_{-}/T_{-} > 1$.

• If $T_{-} < 0$, the condition is given by

$$\sqrt{1+\omega\frac{D-2}{D-1}} > -\frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}.$$
(26)

The solution is $\gamma > 1/(D-1)$ or $\omega > \omega_{\kappa}$ for $\gamma < 1/(D-1)$. Therefore, the solution is summarized by $\omega > \omega_{\kappa}$. However, in the case $\omega > \omega_{\kappa}$, there is no region satisfying $T_{-} < 0$.

B. $H_+/T_+ > 1$

• For $T_+ > 0$, the condition $H_+/T_+ > 1$ is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} > \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)}.$$
(27)

The solution is $\gamma < 1/(D-1)$ or $\omega > \omega_{\kappa}$ for $\gamma > 1/(D-1)$. Thus the solution is summarized by $\omega > \omega_{\kappa}$, as in the last case. The regions III^+ , VII^+ , and IX^+ have solutions satisfying $\omega > \omega_{\kappa}$, $T_+ > 0$, and $H_+/T_+ > 1$.

• If $T_+ < 0$, the above condition is reduced to

$$\sqrt{1 + \omega \frac{D-2}{D-1}} < \frac{(D-1)\gamma - 1}{(1-\gamma)(D-1)},$$
(28)

whose solution is $\gamma > 1/(D-1)$ and $\omega < \omega_{\kappa}$. Therefore, there is no solution satisfying $\omega < \omega_{\kappa}$ because the solution $\omega < \omega_{\kappa}$ is inconsistent with the assumption $\omega > \omega_{\kappa}$.

C. The power behavior of the scale factor as $\tau \to 0$

As
$$\tau \to 0$$
, $a(t)$ is given by
 $a(t) \approx E \times |t|^{\Gamma}$
(29)

where Γ is given by

$$\Gamma = \frac{2(1-\gamma)(\omega - \omega_{\nu})}{(\eta - 1)\kappa},$$

and the constant E becomes

$$E = \left[q(\eta-1)\right]^{\frac{2(1-\gamma)(\omega-\omega_{\nu})}{(\eta-1)|\kappa|}} e^{B\left[1-\frac{2(D-1)\gamma(1-\gamma)(\omega-\omega_{\nu})}{(\eta-1)\kappa}\right]}$$

For $\omega > \omega_{\kappa}$ and $\eta > 1$, the condition that Γ is positive is satisfied in the region $\omega > \omega_{\nu}$. The condition $\Gamma > 1$ is reduced to

$$(1 - \gamma) + [(D - 1)\gamma + (D - 3)]\omega + (D - 2) < 0.$$
(30)

This gives the following solution:

$$\omega < -\frac{D-2}{(1-\gamma)((D-1)\gamma + D-3)} \quad \text{for } \gamma > -\frac{D-3}{D-1}, (31)$$

$$\omega > -\frac{D-2}{(1-\gamma)((D-1)\gamma + D-3)} \quad \text{for } \gamma < -\frac{D-3}{D-1}.$$
(32)

Note that for $\gamma > -\frac{D-3}{D-1}$, ω_{κ} is always greater than $-\frac{D-2}{(1-\gamma)((D-1)\gamma+D-3)}$. Thus in that case, there is no solution. As a result, the solution to $\Gamma > 1$ is given by Eq. (32). This divides region XI of Fig. 1 into two region: for $0 < \Gamma < 1$ we call this as region XI and for $\Gamma > 1$ we call this as region XI and for $\Gamma > 1$ we call this as region XI. See Fig 2. Now we summarize all possible phases in Table 2.

Using the numerical work, we show the explicit scale factor behaviors of all phases in the following figures. Here, we set D = 4. All phases in Fig. 3 have no singularity at $\tau = 0$, and the phases II and IV have no initial singularity. The asymptotic behavior of the scale factor is determined by H_{\pm}/T_{\pm} . Note that phases III^{-} and III^{+} , which are continuously connected at $\tau = 0$, are not distinguished in Ref. 6. Since the scale factors of phases III^{\pm} vanishe at $\tau = 0$, they are divided into two phases in this paper.

Each region included in Fig. 4 and Fig. 5 has two branches, and each branch defines a different phase. For example, the earlier asymptotic behavior of the VII^{-} phase (Fig. 4(a)) and the later asymptotic behavior of the VII⁺ phase (Fig. 4(b)) are determined by H_+/T_+ , and the later behavior of the VII^- phase and the ear motivated [6] by the string cosmology [11] with a gas of solitonic p-brane by treating it as perfect fluid-type matter in the Brans-Dicke theory. In the Brans-Dicke theory, matter has no dilaton coupling. From the string theory point of view, this means that matter has R-R charge. Hence the matter considered here corresponds to a D-brane gas in string theory context. With this matter, we found exact cosmological solutions for any Brans-Dicke parameter ω and for the general constant γ , and we classified all possible phases of the solutions according to the parameters involved. There are two new phases XII^{\pm} different from XI^{\pm} for the behavior of scale factor at $\tau = 0$. Thus, the number of total phases is nineteen, and some of them have no initial singularity. We studied all the phases of the cosmology numerically and presented figures for the time evolution of the scale factor.

Recently, Ref. 12 argued that the holographic principle in the presence of a cosmological constant might imply the absence of an initial singularity. In the Brans-Dicke cosmology, we do find some solutions avoiding the initial singularity. However, when we regard the Brans-Dicke theory as a string cosmology, we might ask whether there are solutions which resolve the initial singularity and the graceful-exit problems at the same time. However, the cosmological constant in the Brans-Dicke theory is not a cosmological constant in string theory where the cosmological constant couples to the dilaton. To discuss the problem in our framework, we have to consider matter coupled with the dilaton. We will discuss this problem in later publications.

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REFERENCES

- [1] E. Witten, Nucl. Phys. B460, 335 (1996), hepth/9510135
- J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); J.
 Polchinski, S. Chaudhuri and C. V. Johnson, hep-th/9602052; E. Witten, Nucl. Phys. **B460**, 335 (1996).
- C. H. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961);
 H. Nariai, Prog. Theor. Phys. **42**, 544 (1968); M. I. Gurevich, A. M. Finkelstein and V. A. Ruban, Astrophys. Space Sci. **22**, 231 (1973)
- [4] M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rep. 259, 213 (1995), hep-th/9412284.
- [5] K. Rama, hep-th/9701154. For earlier graviton-dilaton models, see hep-th/9608026 and hep-th/9611223.
- [6] Chanyong Park and Sang-Jin Sin, Phys. Rev. D57, 4620 (1998).
- [7] Sung-geun Lee and Sang-Jin Sin, J. Korean Phys. Soc. 32, 102 (1998).
- [8] Jai-chan Hwang, J. Korean Phys. Soc. 28, S502 (1995);
 Sung-Won Kim, Sang Pyo Kim, Joohan Lee and D. H. Park, J. Korean Phys. Soc. 28, S519 (1995).
- [9] S. Weinberg, Gravitation and Cosmology (Wiley, New York, 1972).
- [10] G. Veneziano, Phys. Lett. **B265**, 287 (1991).
- [11] G. Veneziano, hep-th/9510027, for string cosmology, there is a vast number of references. Here, we list some of the relevant ones; for more references, see Mod. Phys. Lett. A8, 3701 (1993), and the references therein. G. Veneziano, Phys. Lett B265, 287 (1991); M. Gasperini, J. Maharana, G. Veneziano, hep-th/9602087; Soo-Jong Rey, hep-th/9605176; Nucl. Phys. Proc. Suppl. 52A, 334 (1997); hep-th/9609115; M. Gasperini and G. Veneziano, hep-th/9607126; R. Brustein and G. Veneziano, Phys. Lett. B329, 429 (1994); E. J. Copeland, A. Lahiri and D. Wands, Phys. Rev. D50, 4868 (1994); H. Lu, S. Mukherji and C. N. Pope, hep-th/9610107; A. Lukas, B. A. Ovrut and D. A. Waldram, Phys. Lett. B393, 65 (1997), hep-th/9608195, hep-th/9610238, and hep-th/9611204. S. Mukherji, hep-th/9609048.

-470-

[12] Dongsu Bak and Soo-Jong Rey, hep-th/9811008; G.
 't Hooft, Dimensional Reduction in Quantum Gravity

(World Scientific, Singapore, 1993), p. 284; L. Susskind, J. Math. Phys. **36**, 6377 (1995).