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Condensation of Localized Tachyons and Spacetime Supersymmetry

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We consider condensation of localized closed string tachyons by examining the recent proposal of Harvey, Kutasov, Martinec, and Moore (HKMM). We first observe that the $g_{\rm cl}$ defined by HKMM does not reflect the space-time supersymmetry when the model has the supersymmetry (SUSY). Especially for $\mathbf{C}^2/\mathbf{Z}_N$ models, $g_{\rm cl}$ is highly peaked along the supersymmetric points in the space of orbifolds, which is an unsatisfactory property of the c-function of the renormalization group(RG)-flow. We give an alternative proposal for the tachyon potential in type II cases such that it has a valley along the supersymmetric points in the orbifold moduli space. A new definition predicts that the processes suggested by Adams, Polchinski, and Silverstein and argued to be forbidden by HKMM are in fact allowed.

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I. INTRODUCTION

Recently, there has been a great interest in tachyon condensation in closed string theories. Closed string tachyons indicate the decay of the spacetime itself, and a full dynamical understanding is still lacking. To simplify the problem and to utilize the experience from the open string case, Adams, Polchinski, and Silverstein (APS) [1] studied the cases where the tachyons are localized on a submanifold of space-time, *i.e.*, the tip of the orbifolds. They argued that by tachyon condensation, the orbifolds decayed and the tip of the orbifold smoothed out. They supported this conjecture by D-brane probes in the sub-stringy region and by general relativity analysis beyond the string scale. Since at the tip of the orbifold, the dimensionality of the space time is less than full 10 dimensions, the rotation leaves no spinor invariant, so supersymmetry is completely broken. Localized tachyons live in the twisted sector of the orbifold theory. Vafa [2] argued for a similar picture in the context of gauged linear sigma models and their mirror counterparts. Dabholkar and Vafa [3] proposed a closed string tachyon action where the potential term was given by a c-function.

In relation to this decay of the orbifold background into smoother one, a very interesting proposal was put forward by Harvey, Kutasov, Martinec, and Moore (HKMM) [4]. They identified a certain quantity which decreases along the renormalization group (RG) flow, so one may regard this quantity as a "potential" governing the RG flow. From the world sheet point of view, localized closed string tachyons correspond to relevant perturbations in the twisted sector of the world, and the decay induced by the tachyons correspond to world sheet RG flow induced by a perturbation with the tachyon operator. HKMM proposed to use the analog of the boundary entropy g of Affleck and Ludwig [5] and called this quantity $g_{\rm cl}$, which can be obtained from the partition function in the limit $\tau_2 \to 0$, where τ_2 is the imaginary part of the modular parameter τ:

$$Z(\tau_2 \to 0) \sim g_{\rm cl} \exp(\pi c/6\tau_2). \tag{1}$$

In conformal field theories, the central charge decreases along the RG flow [6, 7]. However, when the central charges are not affected by the RG flow, as with boundary perturbations [5,8], the boundary entropy decreases along the RG flow. The motive of HKMM was to introduce a similar quantity in the case of localized tachyons. In this paper, we would like to continue this study of closed string tachyon condensation by examining the proposal of HKMM. Our work is motivated by the observation that the $g_{\rm cl}$ does not reflect the space-time supersymmetry when the model has the supersymmetry (SUSY). In fact, for $\mathbf{C}^2/\mathbf{Z}_N$ models, $g_{\rm cl}$ defined by HKMM is highly peaked along the supersymmetric points in the space of orbifolds. Certainly, this is not a property of the c-function for RG-flow.

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In this paper, we modify the definition of the $g_{\rm cl}$ in type II cases such that it has a valley along the supersymmetric points in the orbifold moduli space. The main technical differences of this paper from Ref. 4 are threefold:

1. Instead of looking at the high temperatures, we look at the low temperatures. We use
$$Z(\tau_2 \to \infty) \sim g_{\rm cl} \exp(\pi c \tau_2/6)$$
to extract information on the process due to localized tachyon condensations. High and low temperatures give the same information for the total degrees of freedom, but for localized (or delocalized) degrees of freedom separately, we get different results [9].
2. Instead of looking at the twisted partition function measuring localized degrees of freedom, we look at the untwisted partition function measuring the delocalized degrees of freedom. This leads us to the g_{cl} which agrees with Ref. 4 for the type 0 case and to a new g_{cl}^{II} for the type II case, which guarantees the stability of supersymmetric models.
3. For the type II case, we use the full partition function rather than its bosonic part. For counting the central charges, that should not make much difference since the bosonic part does not contain the bulk tachyon anyway, but for the purpose of getting g_{cl}, it is important to use the full partition function.

At first look, considering the delocalized degrees of freedom to describe the localized tachyon condensation may look odd. However, in the orbifold case, once the local geometry near the fixed point(s) is determined, the global geometry is also determined. Once the local geometry changes, the global geometry must also change. Thus, it is natural that the dynamics of delocalized degrees of freedom should encode information on the localized degrees of freedom [9].

For technical convenience and for future interest, we consider superstring theory on a Melvin background [10]. There are a few motivations for doing this: (i) it is exactly solvable [11], (ii) it has tachyons in the spectrum, and (iii) most importantly for our purpose, it reduces to orbifolds [12] in a certain limit. Aspects of a Melvin background in M-theory were studied in Refs. 13-15. Tachyon condensation in this background was discussed using a D-brane probe [16] and in the context of a gauged linear sigma model and mirror symmetry [17]. For earlier literature on tachyon condensation in string theory see Ref. 18.

II. PROPOSAL FOR A POTENTIAL FOR RG-FLOW

The relevant orbifold partition function was first evaluated by Dabholkar in Ref. 19. Here, for our technical convenience, we start from the expression of orbifold partition function given by Takayanagi and Uesugi in Ref. 12, which was obtained by taking a limit of the Melvin background partition function first calculated by Russo and Tyestlin [12]. We will follow closely the notations of Refs. 11 and 12.

1. $\mathbf{C}/\mathbf{Z}_N$ Model

Let us consider the path integral formulation of a Green-Schwarz string on a NS-NS(Neveu-Schwarz) Melvin background with $M_3 \times R^{1,6}$. M_3 being given by a S^1 fibration over R^2. The coordinates of R^2 and S^1 are (ρ, ϕ) and y with radius R. In the light-cone Green-Schwarz formulation, we have eight bosonic fields: $\rho, \phi, Y, X_i (i = 2, \cdots, 6)$. We have [11]

$$Z(R,q,\beta) = (2\pi)^{-7} V_7 R (\alpha')^{-5} \int \frac{(d\tau)^2}{(\tau_2)^6} \int (dC)^2 \sum_{w,w' \in \mathbf{Z}} \frac{|\theta_1(\frac{\chi}{2}|\tau)|^8}{|\eta(\tau)|^{18} |\theta_1(\chi|\tau)|^2} \times \exp\left[-\frac{\pi}{\alpha \tau_2} (4C\bar{C} - 2\bar{C}R(w' - w\tau) + 2CR(w' - w\bar{\tau})) \right], \tag{2}$$

where

$$\chi = 2\beta C + qR(w' - \tau w), \quad \bar{\chi} = 2\beta \bar{C} + qR(w' - \bar{\tau} w). \tag{3}$$

In the above w, w' are the winding numbers, q, β are the parameters proportional to the strengths of the two gauge fields of the Kaluza-Klein Melvin background. $C, \bar{C}$ are auxiliary parameters, and $\tau = \tau_1 + i\tau_2$ is the modular parameter. If

$$\theta_3(0 \mid \tau)^3 \theta_3(\chi \mid \tau) - \theta_2(0 \mid \tau)^3 \theta_2(\chi \mid \tau) - \theta_4(0 \mid \tau)^3 \theta_4(\chi \mid \tau) = 2\theta_1 \left(\frac{\chi}{2} \mid \tau \right)^4, \tag{4}$$

and the quasi-periodicity of theta-functions are used, the partition function in Eq. (2) in the limit $R \to 0$ and

$\beta\alpha'/R \to 0$ with the rational value $qR = \frac{k}{N}$ being finite, is given as follows [12]:

$$\lim_{R\to 0} Z(R,q,\beta) = (2\pi)^{-7} V_7 R(\alpha)^{-4} \int \frac{(d\tau)^2}{4(\tau_2)^5} \sum_{l,m=0}^{N-1} \sum_{\alpha,\beta\in\mathbf{Z}} \left(\lim_{R\to 0} e^{-\frac{\pi N^2 R^2}{\alpha\tau_2}|\alpha-\beta\tau|^2} \right) \times \frac{|\theta_3(\nu_{lm}|\tau)\theta_3(\tau)^3 - (-1)^{k\alpha}\theta_2(\nu_{lm}|\tau)\theta_2(\tau)^3 - (-1)^{k\beta}\theta_4(\nu_{lm}|\tau)\theta_4(\tau)^3|^2}{4|\eta(\tau)|^{18}|\theta_1(\nu_{lm}|\tau)|^2}, \quad (5)$$

where $\nu_{lm} = \frac{lk}{N} - \frac{mk}{N}\tau$, and integers $l,\ m,\ \alpha,\ \beta$ are given as $w' = N\alpha + l, w = N\beta + m$ $(l,m = 0,1,\cdots,N-1)$. The case $l = m = 0$, which correspond to the bosonic zero mode, should be excluded.

If k is an odd integer,

$$Z(0,q,\beta) = \frac{1}{2} V_1 V_7 \int \frac{(d\tau)^2}{4(\tau_2)} (4\pi^2\alpha\tau_2)^{-4} \sum_{l,m=0}^{N-1} Z^0_{l,m}, \quad (6)$$

where

$$Z^0_{l,m} = \frac{|\ \theta_3(\nu_{lm}\mid\tau)\theta_3(\tau)^3\ |^2 + |\ \theta_2(\nu_{lm}\mid\tau)\theta_2(\tau)^3\ |^2 + |\ \theta_4(\nu_{lm}\mid\tau)\theta_4(\tau)^3\ |^2}{2N\ |\ \eta(\tau)\ |^{18}|\ \theta_1(\nu_{lm}\mid\tau)\ |^2}, \quad (7)$$

and $V_1 = \lim_{R\to 0} \frac{2\pi\alpha'}{NR}$ corresponds to the volume of the noncompact direction. Thus, the model can be identified with the orbifold $\mathbf{C}/\mathbf{Z}_N$ in type 0 string theory with radius $\frac{\alpha'}{2NR} \to \infty$ [12]. One should notice that the appearance of a type 0 Gliozzi-Scherk-Olive (GSO) projection from the Green Schwarz path-integral is a very surprising phenomenon. It happens only for the orbifold limit and for an odd integer k, which is a measure zero set of moduli space of Melvin geometry. The sums over l in Eq. (6) corresponds to a $\mathbf{Z}_N$ projection (twists in the time direction) and m is over twisted sectors (twists in the space direction), which are required for modular invariance.

Following Harvey *et al.*, we define the untwisted partition function by

$$Z_{un}(\tau) = \sum_{l=0}^{N-1} Z_{l,0} \quad (8)$$

and the twisted partition function by $Z_{tw}(\tau) = Z(\tau) - Z_{un}(\tau)$. Now, we look at the low temperature limit of the untwisted partition function:

$$Z^0_{un}(\tau\to\infty) = g^0_{cl} \cdot \mathrm{e}^{2\pi\tau_2}, \quad (9)$$

where

$$g^0_{\mathrm{cl}} = \frac{1}{N} \sum_{l=1}^{N-1} \frac{1}{(2\sin[\frac{\pi kl}{N}])^2}, \quad (10)$$

which reproduces the result first given by Harvey *et al.* [4] who used a somewhat opposite logic. Apart from reproducing the result, our procedure is more satisfying in the sense that the central charge counts the bulk degrees of freedom rather than the localized degrees of freedom. Since we already explained in the introduction why bulk degrees of freedom contain information on the process caused by localized tachyon condensation, we do not repeat it here. The sum can be explicitly evaluated to give

$$g^0_{\mathrm{cl}} = \frac{1}{12}\left(N - \frac{1}{N}\right), \quad (11)$$

whose monotonicity in N is used to support the conjecture that g^0_{cl} is the potential for the RG-flow [4].

If k is an even integer,

$$Z(0,q,\beta) = V_1 V_7 \int \frac{(d\tau)^2}{4\tau_2} (4\pi^2\alpha'\tau_2)^{-4} \sum_{l,m=0}^{N-1} Z^{II}_{l,m}, \quad (12)$$

where

$$Z^{II}_{l,m} = \frac{|\theta_3(\nu_{lm}|\tau)\theta_3(\tau)^3 - \theta_2(\nu_{lm}|\tau)\theta_2(\tau)^3 - \theta_4(\nu_{lm}|\tau)\theta_4(\tau)^3|^2}{4N|\eta(\tau)|^{18}|\theta_1(\nu_{lm}|\tau)|^2}. \quad (13)$$

The model can be identified with the orbifold $\mathbf{C}/\mathbf{Z}_N$ in type II string theory [12]. These orbifolds include those discussed in Ref. 1 by APS ($k = N + 1$).

For the definition of $g_{\rm cl}$ for type II string theory, HKMM proposed that one should truncate the spectrum to the bosonic sector; namely, one should make following change:

$$(1 + (-1)^{F_L})(1 + (-1)^{F_R})/4 \to (1 + (-1)^{F_L+F_R})/4.$$

The net result is that $g_{\rm cl}$ of HKMM is identical to that of type 0 string theory up to a factor of a half:

$$g^{II}_{\rm cl,HKMM} = \frac{1}{2} g^0_{\rm cl}. \tag{14}$$

Although it has the right properties for many purposes, it has one critical disadvantage; it does not reflect the consequence of space-time supersymmetry of supersymmetric models, which means that $g^{II}_{\rm cl,HKMM}$ predicts that some supersymmetric models can still decay. Therefore, we proceed differently. We simply define $g^{II}_{\rm cl}$ as the leading term of the partition function of delocalized states, $Z_{un}(\tau_2 \to \infty)$ as before. Due to the GSO projection, the leading tachyon contribution cancels out. Notice that although the main tachyon contribution ($\sim \exp(c\pi\tau_2/6)$) cancels out, it is still non-zero. Thus, we define this leading term as $g^{II}_{\rm cl}$, the potential for the RG-flow; namely,

$$Z^{II}_{un}(\tau_2 \to \infty) \sim g^{II}_{\rm cl} q^0. \tag{15}$$

A simple calculation shows that

$$g^{II}_{\rm cl} = \frac{1}{4N} \sum_{l=1}^{N-1} \frac{(2\sin[\frac{\pi k l}{2N}])^8}{(2\sin[\frac{\pi k l}{N}])^2}. \tag{16}$$

Notice that $Z^0_{un}(\tau_2 \to \infty)$ is infinitely larger than $Z^{II}_{un}(\tau_2 \to \infty)$. Since two objects do not share the same infinite factor, it would be better to define $Z_{un}(\tau_2 \to \infty)$ itself as the potential of the RG flow. Under this criterion, there is no RG flow from type II to type 0 orbifold. Especially, the process of decay of a type II orbifold model into another type II orbifold plus a baby universe of type 0, suggested in Ref. 4, seems implausible. Within the type II orbifold models, $g^{II}_{\rm cl}$ and $g^{II}_{\rm HKMM,cl}$ give the same physics because their behaviors are very much the same. Both are monotonic in N, and the leading terms are linear in N. The monotonicity of $g^{II}_{\rm cl}$ can be easily checked using the following identity:

$$\begin{aligned} g^{II}_{\rm cl} &= \frac{32}{3}\left(N + \frac{2}{N}\right) - 30 \ \text{ for } \ N > 1 \\ &= 0 \ \text{ for } \ N = 1. \end{aligned} \tag{17}$$

However, for higher dimensional orbifold models of $\mathbf{C}^2/\mathbf{Z}_N$, we will see that $g^{II}_{\rm cl}$ and $g^{II}_{\rm HKMMcl}$ predict very different physics within type II theory.

2. $\mathbf{C}^2/\mathbf{Z}_N$ Model

We now consider the higher internal dimensional models. We also start from the expression of orbifold partition function in Ref. 12. Here we have a background geometry of $M_5 \times R^{1,4}$ where M_5 is a fibration of S^1 over $R^2 \times R^2$. We now have two gauge fields, A_ϕ and A_θ, whose strengths involve q_1, q_2 and β_1, β_2. Again, we will be considering the orbifold limit of $R \to 0$ with $\beta_i \alpha'/R \to 0$. We also have $q_i R = k_i/N$ ($i = 1, 2$).

We first consider the case where $k_1 + k_2$ is odd. The partition function is given by

$$\begin{aligned} Z(0, q_1, q_2, \beta_1, \beta_2) &= \frac{1}{2} V_1 V_7 \int \frac{(d\tau)^2}{4\tau_2} (4\pi^2\alpha'\tau_2)^{-3} \\ &\times \sum_{l,m=0}^{N-1} \frac{|\theta_3(\nu^1_{l,m}|\tau)\theta_3(\nu^2_{l,m}|\tau)\theta_3(\tau)^2|^2 + |\theta_2(\nu^1_{l,m}|\tau)\theta_2(\nu^2_{l,m}|\tau)\theta_2(\tau)^2|^2 + |\theta_4(\nu^1_{l,m}|\tau)\theta_4(\nu^2_{l,m}|\tau)\theta_4(\tau)^2|^2}{2N|\eta(\tau)|^{12}|\theta_1(\nu^1_{l,m}|\tau)\theta_1(\nu^2_{l,m}|\tau)|^2}. \end{aligned} \tag{18}$$

We have used $\nu^i_{l,m} = \frac{k_i}{N}(l - m\tau)$. This corresponds to the type 0 case. It is straightforward to see that

$$g^0_{\rm cl}(N, k_1, k_2) = \frac{1}{N} \sum_{l=1}^{N-1} \frac{1}{(4\sin[\frac{\pi k_1 l}{N}]\sin[\frac{\pi k_2 l}{N}])^2}. \tag{19}$$

Next, we turn to the case when $k_1 + k_2$ is even:

$$\begin{aligned} Z(0, q_1, q_2, \beta_1, \beta_2) &= V_1 V_5 \int \frac{(d\tau)^2}{4\tau_2} (4\pi^2\alpha'\tau_2)^{-3} \\ &\times \sum_{l,m=0}^{N-1} \frac{|\theta_3(\nu^1_{l,m}|\tau)\theta_3(\nu^2_{l,m}|\tau)\theta_3(\tau)^2 - \theta_2(\nu^1_{l,m}|\tau)\theta_2(\nu^2_{l,m}|\tau)\theta_2(\tau)^2 - \theta_4(\nu^1_{l,m}|\tau)\theta_4(\nu^2_{l,m}|\tau)\theta_4(\tau)^2|^2}{4N\ |\eta(\tau)|^{12}|\theta_1(\nu^1_{l,m}|\tau)\theta_1(\nu^2_{l,m}|\tau)|^2}. \end{aligned} \tag{20}$$

This is the type II case, and we have the following expression:

$$g_{\rm cl}^{II}(N,k_1,k_2) = \frac{1}{4N}\sum_{l=1}^{N-1}\frac{(4\sin[\frac{\pi(k_1+k_2)l}{2N}]\ \sin[\frac{\pi(k_1-k_2)l}{2N}]\ )^4}{(4\sin[\frac{\pi k_1 l}{N}]\sin[\frac{\pi k_1 l}{N}])^2}. \quad (21)$$

Here again, the main tachyonic piece is cancelled out, and we define $g_{\rm cl}^{II}$ as the leading piece of $Z_{un}(\tau_2\to\infty)$ as before. Notice that along $k_1=\pm k_2$, $g_{\rm cl}^{II}$ vanishes and has a valley reflecting the supersymmetry of the model. We can clearly see this in Fig. 1. This guarantees the stability of the supersymmetric models. On the other hand, according to HKMM, we have $g_{\rm cl}^{II}=\frac{1}{2}g_{\rm cl}^0$, which shows a ridge instead of a valley along the supersymmetric points $k_1=\pm k_2$, as we can see in Fig. 2. We give contour plots of $g_{\rm cl}(N=23,k_1,k_2)$ in Figs. 1 and 2. The lower the value of $g_{\rm cl}$, the darker the shade is in the figures.

Another example that is worthwhile mentioning is the example of APS: $\mathbf{C}^2/\mathbf{Z}_{2l(3)}\to\mathbf{C}^2/\mathbf{Z}_{l(1)}$. This was mentioned as a possible counter example of $g_{\rm cl,HKMM}^{II}$ in Ref. 4. Based on the inequality

$$g_{\rm cl}^0(2l,3,1) < g_{\rm cl}^0(l,1,1), \quad (22)$$

HKMM argued that the process should be impossible. However, according to $g_{\rm cl}^{II}$ given in Eq. (21), this process is possible since we now have the following inequality:

$$g_{\rm cl}^{II}(2l,3,1) > g_{\rm cl}^{II}(l,1,1). \quad (23)$$

In fact, $g_{\rm cl}^{II}(2l,3,1) > g_{\rm cl}^{II}(l,1,1)+g_{\rm cl}^{II}(l,-3,1)$ also holds.

3. Large N limit

It may be of some interest to notice that there are well-defined large-N limits of $g_{\rm cl}$ that can be given by integral expressions. For the $\mathbf{C}/\mathbf{Z}_N$ model, there is no k dependence for finite N, so one may expect that it would not be interesting to take the $N\to\infty$ limit. However, it is not so clear whether this is true for the limiting cases, so we write down the expressions here. For $\mathbf{C}/\mathbf{Z}_N$ models, we have

$$g_{\rm cl}^0(k) = \frac{1}{\pi}\int_0^\pi \frac{dx}{(2\sin[kx])^2} \quad (24)$$

and

$$g_{\rm cl}^{II}(k) = \frac{1}{4\pi}\int_0^\pi dx\frac{(2\sin[kx/2])^8}{(2\sin[kx])^2}. \quad (25)$$

For $\mathbf{C}^2/\mathbf{Z}_N$ models, we have,

$$g_{\rm cl}^0(k_1,k_2) = \frac{1}{\pi}\int_0^\pi \frac{dx}{(4\sin[k_1x]\sin[k_2x])^2}. \quad (26)$$

One should notice that this is singular when $k_1=\pm k_2$. The integrand has a $1/x^4$ singularity near $x=n\pi/k<\pi$, which corresponds to an N^3 dependence for finite N. For the type II case, we have

$$g_{\rm cl}^{II}(k_1,k_2) = \frac{1}{4\pi}\int_0^\pi dx\frac{(4\sin[(k_1+k_2)x/2]\ \sin[(k_1-k_2)x/2]\ )^4}{(4\sin[k_1x]\sin[k_2x])^2}. \quad (27)$$

Here also, we can easily see the valley along the lines $k_1=\pm k_2$. We believe that these are related to the generic (non-orbifold) case of the Melvin background.

III. DISCUSSION

In this paper, we examined a recent proposal made in Ref. 4, namely, the interpretation of the high-energy

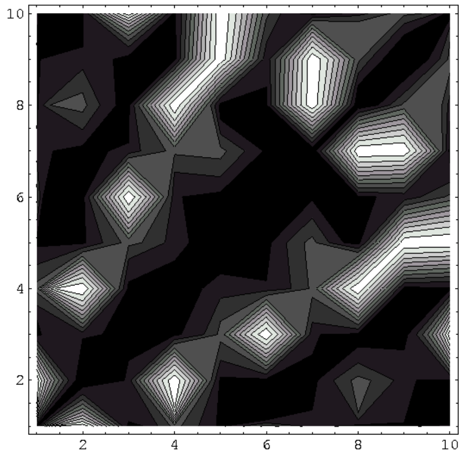

Fig. 1. Contour plot of $g_{\rm cl}^{II}$ in the (k_1,k_2) plane. Only $k_1>0, k_2>0$ is shown. The plot has a valley along the line $k_1=k_2$. Actually the axes correspond to k_i-th prime numbers.

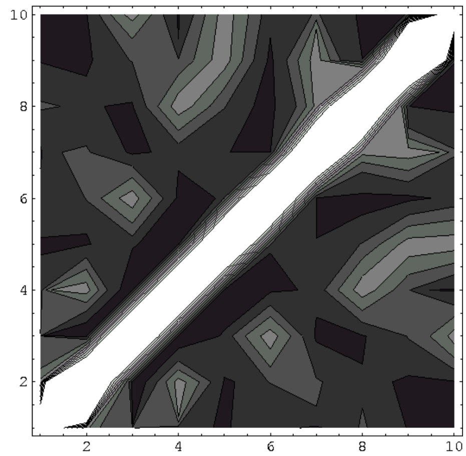

Fig. 2. Contour plot of $g_{\rm cl,HKMM}^{II}=\frac{1}{2}g_{\rm cl}^0$. The plot has a ridge along the diagonal $k_1=k_2$, implying the instability of supersymmetric models.

spectral density $g_{\rm cl}$ as a tachyon potential. We have shown that the $g_{\rm cl}$ defined by HKMM does not reflect space-time supersymmetry when the model has the SUSY. Especially for $\mathbf{C}^2/\mathbf{Z}_N$ models, the $g_{\rm cl}$ defined by them is shown to be highly peaked along the supersymmetric points in the space of orbifolds. This clearly is an unsatisfactory property of the "potential" of the RG-flow. We gave a modified definition of $g_{\rm cl}$ in type II cases such that it has a valley along the supersymmetric points $k_1 = \pm k_2$. Our new definition suggests that the process suggested by APS and argued to be forbidden by HKMM should, in fact, be allowed.

Now, we list some of the issues to be studied in future. One may want to extend the analysis of the D-brane probe and that of HKMM to orbifold models corresponding to $k_2 \neq 1$, which involve orbifolds that go beyond the Hirzebruch-Jung geometry. Another issue is the discussion of the large-N behavior of $g_{\rm cl}$ calculated in this paper. It seems to be related to the generic Melvin background. We wish to come back to these issues in a future publication.

It would be interesting to study this theory in connection to inflection [20], or in diferenct string backgrounds [21].

Finally, there is the issue of the volume factor. Here, we have compared the partition function per unit volume. Since different orbifolds have different volumes, this might affect some of the analysis which have been done here.

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