

# A Gravity Dual of RHIC Collisions

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In the context of the AdS/CFT correspondence, we discuss the gravity dual of a heavy-ion-like collision in a variant of  $\mathcal{N} = 4$  SYM. We provide a gravity dual picture of the entire process by using a model in which the scattering process creates initially a holographic shower in bulk AdS. The subsequent gravitational fall leads to a moving black hole that is gravity dual to the expanding and cooling heavy-ion fireball. The front of the fireball cools at the rate of  $1/\tau$  while the core cools as  $1/\sqrt{\tau}$  from a cosmological-like argument. The cooling is faster than Bjorken cooling. The fireball freezes when the dual black hole background is replaced by a confining background through the Hawking-Page transition.

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## I. INTRODUCTION

The AdS/CFT correspondence [1] has provided a framework for discussing the strongly coupled region of gauge theories in terms of their gravity dual description. The equilibrium finite temperature problem using a black-hole background was discussed in Ref. [2] and the bulk thermodynamics was discussed in Ref. [3]. The transport coefficients [4] in this approach gave a result surprisingly close to what is measured in current heavy-ion collisions at RHIC. Also, the AdS/CFT provides a simple explanation for high energy jet quenching at RHIC [5], an issue of considerable experimental interest in the ‘strongly interacting’ sQGP [6–8]. In fact, following Fermi [9], who first suggested that the collision of strongly interacting matter will produce a thermal state, Landau [10] observed that the system would follow an adiabatic cooling path through transiting thermal states with *entropy* conservation. He further pointed out that the evolution should then be described by (ideal) hydrodynamics. Indeed, one of the key feature of the Quark Gluon Plasma sQGP is precisely the observation of a hydrodynamical expansion in the form of radial and elliptic flow at RHIC.

In Ref. [11], black hole formation in AdS space and its gauge theory dual was discussed using the setting put forward by Polchinski and Strassler [12]. More recently,

in Ref. [13], the authors discussed a scenario leading to a long-lived or quasi-static plasma ball in the same setting. However, high-energy collisions in QCD do not stop the through-going partons.

The purpose of this paper is to provide a general scheme to address the complex issues of thermalization, entropy formation, cooling, and freeze-out in a heavy-ion collision using the gravity dual description of the *time-dependent* black-hole formation. A brief summary of our results was presented in Ref. [14], where we use the AdS/CFT framework along the lines suggested in Ref. [5, 11–13] and [15–18], namely, process of *black-hole formation* followed by a Hawking-Page transition which, from the boundary point of view, corresponds to thermalization, cooling, and finally freezeout through a confinement-deconfinement phase transition. Although the secondary scattering of partons at the boundary is a quantum-mechanical process, its gravity dual is a classical one.

QCD is asymptotically free, and at very short distances, the interaction cannot be strong. Thus, Landau’s scenario can only be applicable after some ‘parton thermalization’ time. In this respect, strongly coupled  $\mathcal{N} = 4$  SUSY YM theory is simpler since it is strong from fiat. Hence, if we can prove that Landau hydrodynamics works in this theory, perhaps with some modifications and corrections, then we can hope to extend the arguments to more QCD-like theories with asymptotic freedom and chiral-deconfinement transitions.

The rest of the paper is organized as follows: In Section II, we give a step-by-step gravity dual to a heavy-ion

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collision, emphasizing the falling of closed strings created by the scattering before the creation of a large black hole. We argue that the black-hole formation is an inevitable consequence of the falling in AdS space. We suggest that the cooling rate of the initial stage of the fireball relates directly to the falling rate of the black hole in AdS. In Section IV, we describe the late-stage cooling of the fireball by using the idea of brane cosmology. Our conclusions and discussions are in Section V. In the appendices, sketches of various ideas for future developments are drawn. In Appendix A, we give a brief summary of some RHIC experimental facts for readers unfamiliar with these current experiments. In Appendix B, we comment on the recently suggested plasma ball. In Appendix C, we discuss collision geometries and entropy production, and give some estimates for physical quantities, such as the saturation scale, in terms of AdS black-hole parameters. In Appendix D, we discuss temperature gradients as corrections to the homogeneous expansion described in Section IV.

## II. RHIC COLLISION AND DUAL BLACK HOLE FORMATION

Recently, two of us suggested [5] that real-time dynamics, such as jet quenching in RHIC, has a gravity dual description in the form of a gravitational wave falling on the black hole. The opacity length was found to be independent of the jet energy at strong coupling [5]. In a related, but different, picture, Nastase suggested that 5D black holes are formed through gravitational collisions of shock waves [18], following on the original work of t'Hooft in flat [19] and Giddings in AdS [11]. Aharony, *et al.* [13] suggested the black-hole dual of static plasma balls in the pure SYM context. It would be interesting to describe the dynamical process by which such object, if any, is formed.

Before proceeding further, state the following:

1. A static black hole is assumed to be formed at once in the IR region [13, 18]. In heavy-ion collisions, the fire ball takes a time (albeit short) to form. To describe the formation and evolution process relevant to thermalization and cooling, the static approach is not appropriate.
2. The assumption of a fixed temperature for the resulting black hole [18] is unrealistic. In heavy-ion collisions, there is no fixed temperature for the fireball; instead, it undergoes an adiabatic path in the phase diagram.
3. The heavy-ion collision involves fundamental (quark) probes that are embedded in the UV region [15, 16], as opposed to the adjoint (glueball) probes set in the IR region [13].
4. The bulk of the thermalization in a heavy-ion collision follows the thousands of elastic collisions, each transferring energy of order  $N_c^0$ , as opposed to a full stopping of energy of order  $N_c^2$  as in Ref. [13].
5. The arguments presented in Ref. [18] and [13] apply equally to proton-proton collisions for which there is no evidence of hydrodynamics behavior, a hallmark of a large black hole.

The scattering process in the boundary occurs with definite energy. How does this translate in the gravity dual space?

### 1. Where is the Holographic Image of Initial Scattering?

In Ref. [12], Polchinski and Strassler argued that the gauge theory scattering amplitude is dominated by a contribution in the dual picture stemming from the height  $r_{scat} \approx \sqrt{s}$ . For definiteness, we give a brief review [12].

Consider the exclusive process  $2 \rightarrow m$  particles. For the gauge theory momentum  $p^\mu$ , we associate the string theory momentum in bulk  $\tilde{p}^\mu$  set with the height  $r$  through <sup>#1</sup>

$$\sqrt{\alpha'} \tilde{p}^\mu = \frac{R^2}{r} p^\mu. \quad (1)$$

The gauge theory amplitude  $A(p)$  at the boundary and the string theory amplitude in a flat space  $A_s(\tilde{p})$  are related by the postulated formula

$$A(p) = \int dr d\Omega_5 \sqrt{g} A_s(\tilde{p}) \prod_{i=1}^{m+2} \psi_i(r, \Omega). \quad (2)$$

The string amplitude  $A_s(\tilde{p})$  fall off exponentially for small  $r$ , and the wave functions fall off at large  $r$  so that the maximum contribution occurs at a finite height,

$$r_{scat} \sim R^2 p. \quad (3)$$

More explicitly, for  $m = 2$ ,

$$r_{scat} \sim R^2 \sqrt{|t| \ln(s/|t|) / (\Delta - 4)}. \quad (4)$$

If  $r_{scat}$  is not smaller than the IR cut-off  $r_{min}$  (the position of the IR brane) the image of the collision in bulk is located at a certain height in AdS rather than the bottom as is claimed in Ref. [13] and [18]. The higher  $\sqrt{s}$ ,

<sup>#1</sup> Here we change the scale of the red-shift factor from the convention of Ref. [12] for later convenience. In our convention, the gauge theory string tension is  $\hat{\alpha}' = \Lambda^{-2}$ , with the minimum height  $r_{min} = R^2 \Lambda$  being defined by the minimum glueball mass  $\Lambda$ .

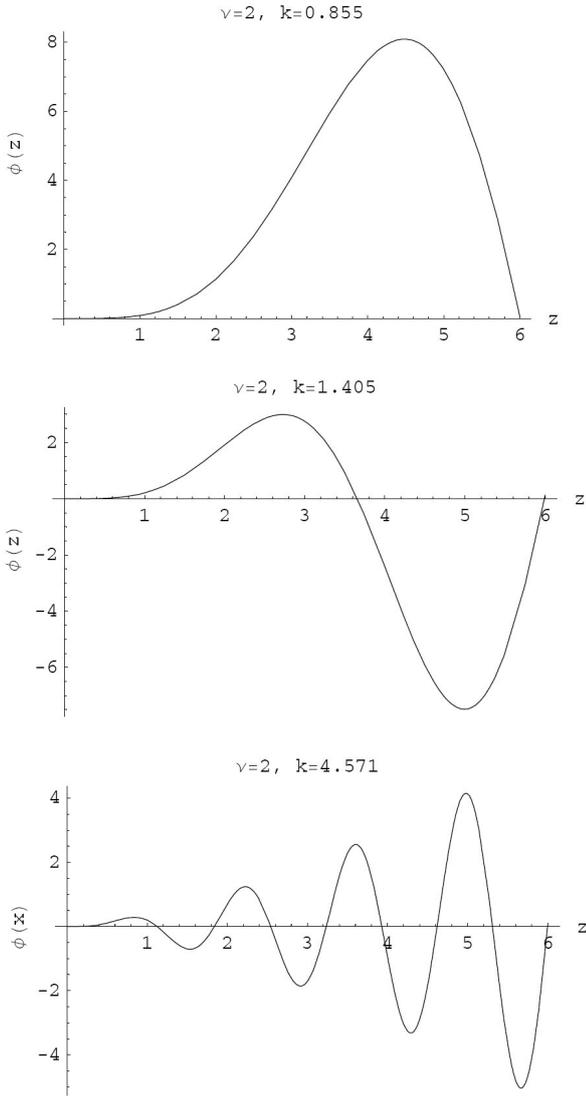


Fig. 1. Wave functions of a few low-energy glueball spectra.

the closer to the boundary #2.

How is this consistent with the fact that the AdS wave-

#2 The image, although localized at  $r_{scat}$  is not sharp unless the object has high conformal weight, for which the size of the holographic image along the  $r$  direction,  $\delta r_{scat}$ , is estimated to be

$$\frac{\delta r_{scat}}{r_{scat}} \sim \frac{1}{\sqrt{\Delta}}. \tag{5}$$

Therefore, for a high conformal weight in 4D, the holographic image is localized. To simplify the discussion, let's discuss the scattering in pure SYM without quarks. If we model the initial beam as a highly excited glueball, then the conformal weight  $\Delta = \sum_{i=1}^{m+2} \Delta_i$  can be considered to be large to mimic a heavy ion-like object. This means that the holographic image of the incoming beam has a well-defined height and the debris of the scattering should fall under AdS gravity. Below we will argue that the debris forms a receding black hole within a dynamical time of order  $\Lambda_{QCD}^{-1}$  at the bottom or IR brane.

functions of the incoming particles are peaked in the IR not the UV, as shown in the Fig. 1? The answer lies in the measure factor. Since the discussions so far did not introduce fermions (fundamental fields), we model the heavy-ion collision as a collision of two highly excited glueball states. For an incoming glueball with definite energy, the scalar wave function is factorized as  $\Phi(x^\mu, z) = e^{ik \cdot x} \phi(z)$ , and the scalar field equation in 5D,  $(\square_5 - m^2)\Phi(x^\mu, z) = 0$ , reduces to

$$[z^5 \partial_z z^{-3} \partial_z - k^2 z^2 - (mR)^2] \phi(z) = 0, \tag{6}$$

with  $k^2 = \vec{k}^2 - \omega^2$ . Therefore, for a process with definite energy-momentum, the wave function depends only on the mass, not on the energy. For a confining theory, we need to cut off the IR part by hand (restrict to  $z > z_m$ ) or by a pertinent metric structure; hence, we are interested in a wave function that is regular near the boundary ( $z \sim 0$ ), where the wave function behaves as  $\phi \sim z^{2\pm\nu}$  with  $\nu = \sqrt{4 + (mR)^2}$  and  $R^4 = 4\pi g_s N \alpha'^2$ . The non-normalizable part ( $z^2 K_\nu(kz), z^2 N_\nu(kz)$ ) should be interpreted as the two-point function with a source at the boundary [25], so that its coefficient is the strength of the source. For the initial beam, it is on-shell and  $k^2 < 0$  is the 4D mass. The explicit normalizable wave function is [24]

$$\phi(z) = z^2 J_\nu(kz) \text{ for } k^2 < 0. \tag{7}$$

On the other hand, between the collisions, the particles are off mass-shell ( $k^2 > 0$ ), so  $k$  should be interpreted as the momentum transfer. The wave function is

$$\phi(z) = z^2 I_\nu(kz) \text{ for } k^2 > 0. \tag{8}$$

As the 5D mass (equivalently  $\sim \nu$ ) increases, no qualitative change in the wave functions is observed, except that it is slightly pushed to the IR region (larger  $z$ ). On the other hand, increasing the 4D mass ( $k^2$ ) includes more nodes in the allowed region and effectively pushes the wave function into UV region. This ‘push-to-UV’ effect is more dramatic if we consider the ‘radial’ probability density  $P(z) = \sqrt{g} |\phi(z)|^2$ . Due to the measure ( $\sqrt{g}$ ), the dominant peak is near the boundary rather than the horizon. We can estimate the location of the dominant peak in terms of  $x_{\nu 1}$ , the first zero of the  $J_\nu(x)$ . We suggest that the location of the holographic image of the incoming glueball with mass  $M_4 (= \sqrt{-k^2} := k)$  is given by

$$r_0 \sim \frac{1}{2} \frac{R^2}{x_{\nu 1}} M_4 \sim R^2 k, \tag{9}$$

which is consistent with Eq. (3).

In summary, if we model a heavy ion as a glueball with a large 4D mass, the holographic image of the initial beam is at a height that is proportional to the mass. This is consistent with Polchinski-Strassler’s argument, as detailed above. The scattering takes place at a given height because their initial states are localized there.

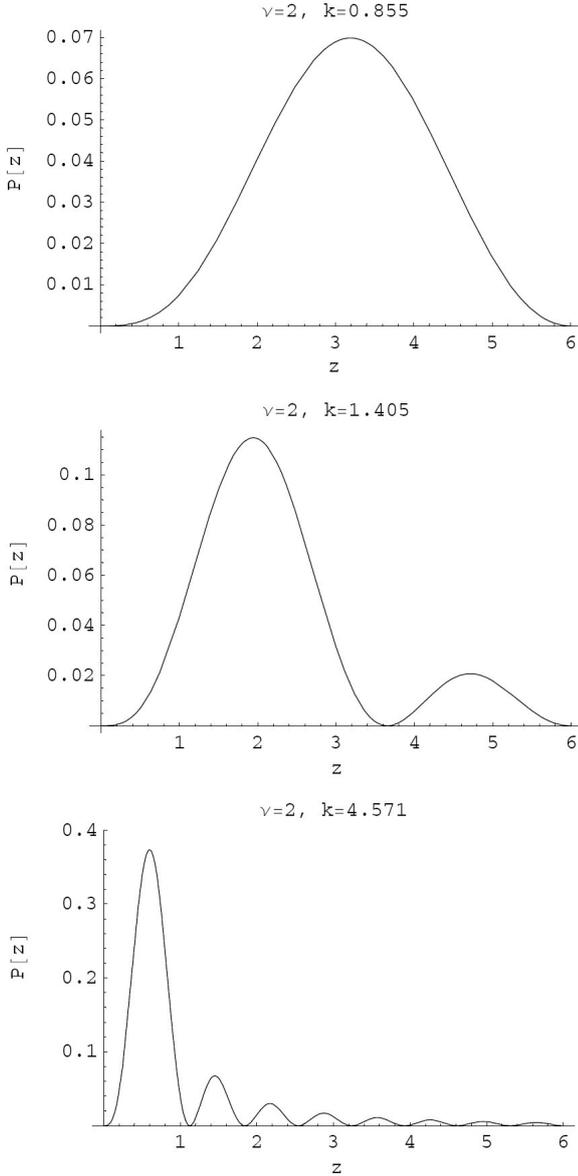


Fig. 2. Probability distributions. Notice that  $z = 0$  is the boundary. For higher excitations it is more likely to be in the UV region.

## 2. Expansion and Thermalization

Since RHIC data shows that the fireball after collision is thermalized, so an AdS black hole should be formed. Then how?. For this, we notice that the AdS gravity has anti-tidal force so that it has a focusing property. Namely, two vertically separated particles in AdS bulk will become closer as they fall. We have a gas of falling debris after collision. If one consider a local rest frame of the fluid (Lagrangian coordinate in fluid mechanics language), the common proper time for all the particles can be used as a time coordinate. This is similar to the treatment of Bjorken in 3+1 spacetime by assum-

ing LRF. As we will see below, all the falling particles released from different heights arrive at the bottom after the same proper time,  $\tau = \pi R/2$ . The unavoidable consequences of this are that the motion of an AdS fluid is like a 'cosmological' contraction leading to a final singularity and that the dual of the fireball forms an AdS black hole. Of course, the dual of this contraction is the fireball expansion in the boundary.

To be more explicit, consider a radial in-fall in AdS space:

$$d\tau^2 = \left(\frac{r}{R}\right)^2 dt^2 - \left(\frac{R}{r}\right)^2 dr^2. \quad (10)$$

For a massless particle, the motion is described by a null geodesic with the solution

$$r = R^2/t, \quad (11)$$

and the falling should start on the AdS boundary at  $t = 0$ , which is consistent with the picture that the free falling of massless particles in AdS is dual to the free expansion in the boundary whose front surface is expanding with the speed of light [5, 20]. For a massive particle, that leads to

$$\left(\frac{dr}{d\tau}\right)^2 + \left(\frac{r}{R}\right)^2 = \epsilon^2, \quad (12)$$

where  $\epsilon = (r/R)^2 \frac{dt}{d\tau}$  denotes the energy per unit mass. The resulting motion is harmonic in proper time,

$$r = R\epsilon \cos(\tau/R), \quad t = R/\epsilon \cdot \tan(\tau/R). \quad (13)$$

The period is  $2\pi R$ , which is *independent of the initial conditions*. In case there is an IR brane, the initial difference in height,  $\delta r(\tau = 0) = R\delta\epsilon$ , will be reduced to  $\delta r(\tau) = R\delta\epsilon \cos(\tau/R)$  at the bottom. In terms of the boundary time  $t$ ,

$$r = \frac{\epsilon R}{\sqrt{(\epsilon t/R)^2 + 1}} = R^2/t - (R/t)^3/2\epsilon^2 + \mathcal{O}(t^{-5}) \quad (14)$$

so that the initial condition dependence (that is the  $\epsilon$  dependence) disappears rapidly as time goes on. We believe that this focusing effect plays an important role in the initial formation of the black-hole geometry. Thus, Eq. (14) can be thought to describe the front surface of a fireball that is not equilibrated.

After reaching bottom (IR region), the *droplet* will spread and flatten to make a pancake. For late time falling objects, such a stack of mass on the IR brane generates a black-hole geometry due to Birkhoff's theorem. See Fig. 3 and its capture. The particles inside the front surface experience an interaction with the medium, and the expansion in the center of the fireball is dual to the falling of a particle in the AdS black-hole background. In the next subsection, we will consider the case with fundamental fields (quarks).

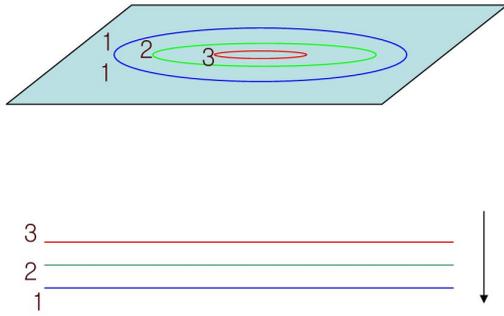


Fig. 3. Holographic correspondence of the expansion in 4d and the falling in 5d. From the boundary point of view, the front part ‘1’ is freely streaming while the inner part ‘3’ sees medium effects. From the bulk point of view: the lower part ‘1’ falls freely while the upper part ‘3’ sees the AdS black hole geometry. Birkhoff’s theorem tells that whether the inner part is really black hole or not is not an issue. Thus the inner part ‘3’ feels that it is in thermal equilibrium.

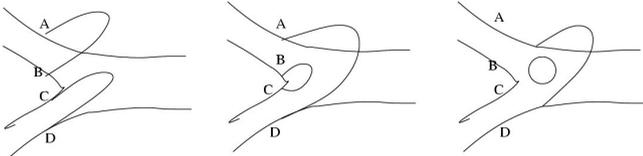


Fig. 4. At each interaction vertex of two scattering mesons a closed string must pop up. This is a unique feature of AdS space that does not take place easily in flat space.

**3. With Quarks: Creation of a Closed String**

If we have particles in the fundamental color representations in addition to the ones in the adjoint color representation, we need to introduce probe branes in the bulk [21]. A heavy meson is a quark and antiquark connected by a string deep in AdS. The scattering of such mesons could be realized by moving these AdS strings, which are highly non-local objects in the a AdS bulk. The collision of such strings in the bulk may form a highly distributed object in the bulk and may not be a black hole initially. However, the contraction and AdS fall of these objects will give a black hole.

The mesons in  $N = 4$  theory were studied in Ref. [17]. They are deeply bound with mass,

$$M \sim \frac{m_q}{\sqrt{N g_{YM}^2}}. \tag{15}$$

Thus, if we model a heavy ion by this meson, the quark mass should be taken as large. In this case, the holographic image of the fireball is created at a significant height ( $\sim M_q$ ) and falls to form a black hole at the bottom. In this picture, we can argue that for each vertex, a closed string can be created to leave behind a flavor brane.

The many-parton collisions at the boundary trigger (i) elastic collisions which are dual to massive closed strings

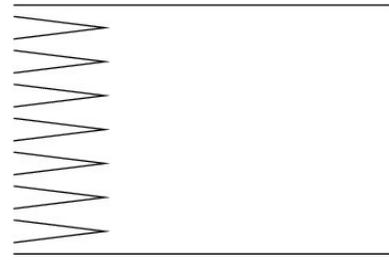


Fig. 5. Multiple interaction vertices create a shower of massive closed strings in AdS space. Some of them are mini-black holes. The strings flake and fall towards the AdS center like a rain-fall to form a large black hole at bottom.

and (ii) inelastic collisions which are dual surface flips. An example of the former process is shown in Fig. 4. Since the minimal string is not a straight line connecting two sources on the boundary (infinite warping factor), the string must stretch inside the AdS space [26,27]. As two mesonic composites come together, a recombination from  $AB + CD$  to  $AD + BC$  should happen just before  $B$  and  $C$  touch each other, because that is energetically favored. For example, when the separation (in boundary) of  $AB$  and  $BC$  are both  $L$  and that of  $BC$  is  $\epsilon$ , then for small enough  $\epsilon$ , the difference in total lengths of the string is

$$l_{AB} + l_{CD} - l_{AD} - l_{BC} = -2\frac{c}{L} + \frac{c}{2L + \epsilon} + \frac{c}{\epsilon} > 0, \tag{16}$$

where  $c$  is just a constant. Thus, in a hadron-hadron scattering process, a recombination of the string must arise at the vertex (where  $B$  and  $C$  coincide), generating a closed string. This is a remarkable feature of AdS space with no analogue in flat geometry. Although the above example is for pure AdS, we expect the mechanism to be universal regardless of the geometry in the IR region if the UV region remains AdS.

Each of these liberated closed strings fall in AdS space under AdS gravity. Some of the closed string states could be in a black-hole state. They merge as they fall to form a larger black hole in the IR region, as we suggested earlier. In fact, the initial colliding objects contain multitudes of particles, and they cannot be in thermal equilibrium at once. Therefore, in the gravity dual that which forms immediately after the collision is not a single big black hole, but a multitudes of mini black holes together with other closed string states.

The efficient creation of the particles provides a mechanism to convert the deposited collision of kinetic energy to mass, lowering the temperature scale, and most of the initial energy is deposited as mass. In real QCD, this procedure increases the strength of the interaction by running coupling. Here, in  $N = 4$  SYM, there is no such effect because coupling does not run. From boundary language, the increase in the number density of particle and the increase in the interaction strength are the key

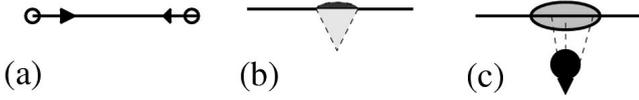


Fig. 6. Schematic view of wall-wall collision. The vertical coordinate is the 5th dimension  $r$ , the horizontal one is the longitudinal one. (a) before the collision, (b) shortly after the collision: color rearrangements and massive production of closed strings. They fall into AdS space; (c) A larger black hole is formed at the bottom. It will flatten to a pancake shape, lowering its horizon.

points for efficient thermalization.

The gravity dual of the collision processes is shown schematically in Fig. 6. Even though matter is partially stopped on the boundary (UV region), the initial entropy build up will cause the formation of a fireball and its expansion. Its gravity dual is a detached set of closed string states that fall into the AdS space.

#### 4. Final Stage: Hadronization

As time goes on, the black-brane-like object becomes thinner and eventually unstable to density fluctuations that lead to an instability [22]. We expect the extremely thin black pancake to fragment into small pieces, each of which evaporates (quantum mechanically) by Hawking radiation. This may be identified as the confinement phase transition. Since the resulting metric is nothing but the AdS metric with an IR cut-off, which is a confining metric, this can be considered as the details of the Hawking-Page transition [2, 17].<sup>#3</sup> In our picture, we do not expect quasi static black holes as in Ref. [13]. This is so in heavy-ion collisions. also because since not all partons are stopped at once, the initial expansion is one dimensional, instead of three dimensional as Bjorken suggested [23]. After local equilibration is achieved in a heavy-ion collision, the matter expands, and the temperature depends on both the location and time. With these considerations, the gravity dual of the RHIC collision can be set up by considering the physical process together with the general dictionary of ADS/CFT listed below.

### III. COOLING OF THE CORE OF THE RHIC FIREBALL

In this section, we discuss cooling and expansion of the fireball in the late stage, where the black hole becomes

<sup>#3</sup> In pure  $N = 4$  SUSY, the interaction is the same at all scales. Thus, an expanding fireball of a ‘‘CFT plasma’’ will never freeze-out and will expand hydrodynamically forever till zero temperature is reached. Freezeout can be reached in the gravity dual by switching to confining D-brane metrics, which is known as a Hawking-Page transition.

a black-brane-like object that is expanding in spatial direction and lowering its horizon continuously under the influence of AdS gravity. The temperature decrease is interpreted as an increase in the distance between the probe brane and the black hole horizon.

Since the AdS space is homogeneous, we can change the frame such that the black brane is fixed while the probe brane is moving in the background of the static black brane. Then, the probe brane moves in the UV direction and, therefore, sees a bigger scale factor of the bulk metric as it moves, resulting in a cosmic expansion on the brane world. This is nothing but the brane world cosmology addressed in Ref. [29–34]. In this way, we identify the gravity dual of the expansion/cooling of the fireball as the cosmic gravitational expansion in the AdS-black-hole background. In other words, we approximate the Little Bang as the Big Bang on the probe brane. In the Little Bang, the temperature has space and time dependence while in the Big Bang there is no spatial dependence, only time dependence. Therefore the approximation is good only for the center of the fireball, which is the subject of this section.

Although the real expansion is mostly one dimensional, we believe that the thermally equilibrated expansion is three dimensional in nature. This is because the one dimensional expansion is driven by the ultra-relativistic motion of the initial particles whose speeds cannot be caught up by the interactions.

#### 1. Big Bang on a Moving Brane

We consider a class of metrics given by the near horizon-limit of non-extremal  $D_p$  branes:

$$ds^2 = g_{00}dt^2 + g(r)d\vec{x}_p^2 + g_{rr}(r)dr^2 + g_S d\Omega_{8-p}, \quad (17)$$

where  $g = (r/R)^{(7-p)/2}$ ,  $|g_{00}| = (r/R)^{(7-p)/2}(1 - (b/r)^{7-p}) = g_{rr}^{-1}$ , and  $g_S = r^2(R/r)^{(7-p)/2}$ . The dilaton is given by

$$e^{2\phi} = \left(\frac{R}{r}\right)^{(7-p)(3-p)/2}. \quad (18)$$

If we neglect the brane-bending effect and consider the configuration of zero angular momentum of the brane around a sphere, the DBI action for the  $D_p$  brane,

$$S_p = -T_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det \gamma_{\alpha\beta}} - T_p \int C_{p+1}, \quad (19)$$

can be written as

$$S_p = -T_p \int d^{p+1}\xi e^{-\phi} g^{p/2} \sqrt{|g_{00}| - g_{rr}\dot{r}^2}. \quad (20)$$

Since there is no explicit time dependence,

$$E = p \cdot q - L = \frac{g^{p/2}e^{-\phi}}{\sqrt{|g_{00}| - g_{rr}\dot{r}^2}} - C, \quad (21)$$

Table 1. Vocabulary of dual phenomena in gauge and gravity formulations.

gauge $d = 4$ theory	string/gravity in $d = 10$
Total cm energy per unit mass	height of scattering brane in AdS
initial gluons, CGC	Aichelburg-Sexl-type shock waves
Thermalization $\rightarrow T$	black hole formation with $T_{bh} = T$
the entropy	area of the horizon
rescattering ( $q - q$ )	production of closed string (gravitons)
rescattering ( $g - g$ )	interactions between closed strings
fireball expansion before equil.	falling b.h. in AdS bulk
fireball expansion after equil.	spreading of b.h at bottom
further equilibration	merger of gravitons to black hole.
ideal hydrodynamics	stationary black hole
hydro with viscosity	growing black hole
kinetic freezeout	cutoff of gravitons
deconfinement	fragmentation of thin black-brane followed by hawking evaporation

with  $C = (r/R)^{7-p}$ , is a constant of the motion. Using the equation of motion

$$g_{rr}\dot{r}^2 + g_{00} + g^p|g_{00}|e^{-2\phi}/(C + E)^2 = 0, \quad (22)$$

the induced metric can be written as

$$ds^2 = -\frac{g_{00}^2 g^p e^{-2\phi}}{(C + E)^2} dt^2 + g dx^2. \quad (23)$$

Defining the proper (cosmic) time  $\tau$  by

$$d\tau = |g_{00}|g^{p/2}e^{-\phi}/(C + E)dt, \quad (24)$$

the induced metric can be written as a zero curvature Friedman-Robertson form,

$$ds^2 = -d\tau^2 + a^2(\tau)dx^2, \quad (25)$$

where  $a^2 = g(r(\tau))$ . The equation of motion can be rewritten in terms of  $a$  and  $\tau$  as

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{(C + E)^2 e^{2\phi}}{|g_{00}|g_{rr}g^p} - \frac{1}{g_{rr}}\right) \left(\frac{g'}{2g}\right)^2, \quad (26)$$

with  $g' = dg/dr$ . Then, the equation of motion in terms of  $a$  and  $\tau$  is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{7-p}{4}\right)^2 a^{2(3-p)/(7-p)} \times \left[\left(\frac{E}{a^4} + 1\right)^2 - \left(1 - \frac{b^{7-p}}{R^{7-p}} \frac{1}{a^4}\right)\right], \quad (27)$$

where we have used the fact  $C = (r/R)^{7-p} = a^4$ . Notice that the effect of the RR-flux field  $C$  is to provide a strong enough repulsion force to cancel the confining AdS gravity.

As  $a \rightarrow \infty$  (late evolution), we have

$$a(\tau) \approx \tau^{(7-p)/(11-p)}. \quad (28)$$

The scale factor evolution  $a(\tau)$  captures the cooling of the fire-ball at the boundary through its holographic dual:

$$T(a) = \frac{T_{bh}}{\sqrt{|g_{00}|}} \approx \frac{T_{bh}}{a(\tau)}, \quad (29)$$

with

$$T_{bh} = \frac{(7-p)}{4\pi b} \cdot \left(\frac{b}{R}\right)^{(7-p)/2} \quad (30)$$

being the black-hole temperature. The local temperature is the black-hole temperature observed by the observer in the probe brane. This is the actual temperature of the fireball. As the brane moves away from the black hole, the brane world (the fireball) expands and cools according to  $T(a) = T_{bh}/a$ .

There are two interesting cases:  $p = 3$  and  $p = 4$ . For  $p = 3$ ,

$$a(\tau) \sim \sqrt{\tau}, \quad T \sim \frac{1}{\sqrt{\tau}}. \quad (31)$$

The reason for considering  $p = 4$  is that one of its direction (say  $x_4$ ) in a confining theory is compactified. After the compactification, the  $p = 4$  and  $p = 3$  cases are identical. Without compactification,  $a \approx \tau^{3/7}$ , which is a stronger warping.

This result is to be compared with cooling in D-space. Indeed, the entropy for a (perfect) gas is just  $S \approx T^D V_D$ . For a relativistic d-space hydrodynamical expansion, we expect  $V_D \approx V_{D-d} \tau^d$ . For fixed entropy, the temperature falls like  $T \approx 1/\tau^{d/D}$ . Bjorken 1-space expansion Eq. (31) corresponds to  $D = 3$  and  $d = 1$ ; therefore,  $T \approx 1/\tau^{1/3}$ . Fully 3-space expansion corresponds to  $T \approx 1/\tau$ . The AdS case with  $T \approx 1/\sqrt{\tau}$  is faster than the Bjorken case in 1-space, but slower than perfect hydrodynamical expansion in 3-space. It is like a fractal

with  $d = D/2 = 3/2$ . One may summarize these results by saying that *strong interactions slow down the expansion of the fireball just as gravity does in the dual picture*. We note that since the viscosity is quantum with  $\eta/(S/V_3) \approx \hbar/4\pi$ , its effects are not present in our estimates. Their consideration follows from perturbation theory and is easily seen to delay the cooling.

## 2. Confinement Phase Transition

When  $T$  cools enough such that  $T < \Lambda_{QCD}$ , there must be a Hawking-Page transition [28] and the background metric is replaced by

$$ds^2 = (r/R)^{3/2}(-dt^2 + d\vec{x}^2 + f_2(r)dx_4^2) + (R/r)^{3/2}(dr^2/f_1 + r^2d\Omega_4^2), \quad (32)$$

where  $f_2 = 1 - (r_{KK}/r)^3$  refers to the compactified direction. Witten [2, 17] suggested that the transition to this metric may be interpreted as the confinement/deconfinement phase transition. The equation of motion in the new background can be calculated. Though minor, there are a few differences in the details of the calculation, but rather surprisingly, the final outcome is precisely the same with the substitution  $b \rightarrow r_{KK}$ . For  $p = 4$ ,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{9}{16}a^{-2/3} \left[ \left(\frac{E}{a^4} + 1\right)^2 - \left(1 - \frac{r_{KK}^3}{R^3} \frac{1}{a^4}\right) \right]. \quad (33)$$

As we discussed before, the front factor  $9/(16a^{2/3})$  disappears if  $x_4$  is compactified (which is effectively  $p = 3$ ). The phase transition point in terms of the brane position occurs when the warping becomes  $a_F$  at

$$T(a_F) \approx T_{KK}, \quad (34)$$

where the Kaluza-Klein temperature is given by  $T_{KK} = 3r_{KK}^{1/2}/(4\pi R^{3/2})$ . Thus,

$$a_F = \frac{T_{bh}}{T_F} = \sqrt{\frac{b}{r_{KK}}} \quad (35)$$

One may interpret the phase transition as a hiding of the black hole horizon behind the KK singularity  $r = r_{KK} \approx 1/\Lambda_{QCD}$ . After this phase transition, hadron creation begins, which should be a dual to the Hawking evaporation of a black brane after a Gregory-Lafllame transition, as we discussed at Section III. The fireball ultimately freezes out when the pions decouple.

So far the expansion is homogeneous. This is consequence of the assumption that the background is a black brane. In reality, the temperature has some gradient. To discuss the spatial dependence, as well as the time of the temperature, we need the brane motion in the background produced by an object with a finite extent like a black hole. We touch on this important, but difficult, subject in appendix D.

## IV. DISCUSSION

We close by summarizing. Recent heavy-ion collisions at RHIC have suggested that the released partonic matter produced is strongly interacting in the form of an sQGP. Two of us have argued recently that a good starting point for addressing key issues of the sQGP is  $\mathcal{N} = 4$  SYM at strong coupling. In this paper, we have suggested that the entirety of the RHIC collision process from the prompt entropy release to the freezeout stage can be mapped by duality to black-hole formation and evolution in AdS space. In other words, the RHIC little bang and cooling is dual to a cosmological big bang with a flying black hole as a proceed.

We have provided a simple physical picture of black-hole formation and thermalization from a string-theory point of view. We have suggested that due to the strong interaction of the fireball liquid, the expansion is slower than expected from the ideal gas model. The cooling of the fireball is  $1/\sqrt{\tau}$ , which is slower than the Bjorken 3D cooling  $1/\tau$ . The strong nature of the interaction slows down the expansion rate; hence, the cooling is slower than expected from the Bjorken solution. Cooling freezes when the background is replaced by the confining background through the Hawking-page transition.

There is a clear distinction between coherent parton-parton scattering and an incoherent macroscopic (heavy-ion) collision of large number of partons. In the former, the scattering happens in the IR region and information is conserved, which is a hallmark of quantum mechanics, while in the latter, the scattering happens in the UV region, information is lost<sup>#4</sup>, and entropy is generated, perhaps to its maximal value, typical for local thermal equilibrium. While entropy generation may be traced back to the incoherence due to the many binary scattering in a RHIC heavy-ion collision, it is readily understood in the gravity dual description: Since the particles evaporate from the crowds, the (contracting) core is losing information and becomes a black hole. Although the lost information will come back to the black hole (due to the AdS gravity) and the black hole grows, all the information is hidden inside the horizon. This, we believe, is one of the simplest explanations for entropy production at RHIC.

Non-cosmological-like expansions with realistic fire ball geometries on the boundary are more involved to analyze. We have suggested that their asymptotic stages can be mapped on black-hole perturbation theory, resulting in non-ideal hydrodynamics, from conventional Einstein gravity. We will report on these issues and others in future.

**Note added:** Since submitting our paper, a new paper by Janik and Peschanski [47] appeared. The cooling

<sup>#4</sup> A hypothetical *full experiment* with measuring phases of all thousands of secondaries can still, in principle, recover it.

of the fireball is discussed using the asymptotic solution for the metric induced by the energy momentum stress tensor set on the *probe brane*. In the gravity dual, the black hole's moving horizon reproduces Bjorken's scaling for a one-dimensional expansion, in support to our arguments.

## APPENDIX A: ELEMENTS OF RHIC PHYSICS

This discussion is intended to be elementary to shorten the vocabulary gaps between the string community and the heavy-ion community interested in the gauge-gravity problems through the AdS/CFT correspondence.

**Collision:** Experimentally we use the heaviest (and fully ionized) nuclei (mostly  $Au^{197}$  at RHIC) with as large an energy per nucleon as possible (the relativistic gamma factors is  $\gamma \sim 100$  in the center of mass, and is to be increase further at LHC soon.)

One may ignore the complexities of nuclear physics and QCD evolution and focus solely on the partonic wave function of hadrons or nuclei before the collision. More precisely, as coherence is lost anyway, one needs to know the mean *squared* amplitudes of the pertinent harmonics of the comoving gluon field with the so called saturation scale  $Q_s$  or equivalently the transverse density of partons  $Q_s^2$ . At RHIC  $Q_s$  is about 1.5 GeV for a typical Feynman  $x = 10^{-2}$ . It will be higher at LHC say  $Q_s = 6 - 8$  GeV at lower  $x$ . A model currently used to describe the low- $x$  part of the nuclear wavefunction prior to the RHIC collision is the color glass condensate (CGC). It is rooted in the weak coupling argument in QCD, contrary to what is stated in Ref. [18].

**Equilibration:** This is a transition from the CGC to thermal quarks and gluons. Classical Yang-Mills, solutions both for random fields [35] and sphalerons [36] have actually produced thermal-looking spectra but more is to be understood, perhaps along the discussion of plasma instabilities [37].

**Hydrodynamics:** This is a key aspect of RHIC physics. Maintaining collective flow for systems containing just  $\sim 100 - 1000$  particles is a nontrivial issue [38], and would not happen for usual liquids like water. Thus, the matter produced at RHIC is now referred to as a strongly coupled quark-gluon plasma (sQGP) or liquid. Indeed it exhibits both bulk thermodynamical parameters and transport coefficients (viscosity) that are surprisingly close to what the AdS/CFT correspondence predicted for strongly coupled  $N = 4$  SUSY YM theory. The short time behavior of the hydrodynamical expansion is close to the 1-d Bjorken region where the temperature depends only on the proper time  $\tau = \sqrt{t^2 - z^2}$ . For central collisions, the expansion becomes axially symmetric before turning to a full 3D spherical expansion. For non-central collisions there is azimuthal anisotropy which is successfully described by hydrodynamics.

**Freezeouts:** This corresponds to chemical and ther-

mal freezeouts, where the change in the composition is turned off (chemical) and the particles decouple (thermal) with free streaming. Both freezeouts follow from the same condition  $\nu_{expansion} = \nu_{reaction}$ , where we have used the covariant definition of the expansion rate  $\nu_{expansion} = \partial_\mu u^\mu$ .

In cosmology, the expansion is so slow that not only does strong ( $pp$ ) scattering survive, but even weak equilibrium through  $p + e \leftrightarrow \nu + n$  does, until  $T \approx 1$  MeV. Photons freezeout at much lower temperatures of  $T \approx 0.1$  eV. At RHIC chemical freezeout corresponds to the end of particle changing reactions such as  $2\pi \rightarrow 4\pi$  while kinetic freezeout corresponds to the last elastic collision such as  $2\pi \rightarrow 2\pi$ . Experimentally, both freezeouts are reasonably well measured, the former from matter composition and the latter from particle spectra [39]. While the critical temperature in QCD,  $T_{ch} \approx 176$  MeV, is independent of the collision centrality, the freezeout temperatures depend on the system size. For instance, the kinetic freezeout temperature  $T_{kin}$  depends on the system size and goes down for the largest fireballs (central collisions) to about 90 MeV. Thus, the whole range of temperatures at RHIC is about 4-fold, from the initial  $T_i \approx 350$  MeV to the kinetic freezeout  $T_{kin} \approx 90$  MeV. The energy density changes by about 2 orders of magnitude.

The main reason for the rapid freezeout of a hadronic gas is the Goldstone nature of the pions. The self-interaction through derivatives makes it difficult to generate soft pions. At low temperature, the pion gas collision rates can be calculated from the leading chiral interaction (Weinberg-Tomozawa). Specifically, the elastic rate is [40]

$$\nu_{\pi\pi} = \frac{T^5}{12f_\pi^4}. \quad (A1)$$

The strong T dependence follows from dimensional arguments. The inelastic rates can be found in Ref. [41].

At RHIC, detailed numerical calculations show that the proper time spent in the sQGP phase ( $T > T_c$ ), the "mixed phase" ( $T \approx T_c$ ), and the hadronic phase ( $T < T_c$ ) are all comparable. However, at LHC, the sQGP should dominate. For simplicity, we may ignore the complications inherent to the running coupling in QCD, the confinement-deconfinement transition, and the pion dynamics by restricting the discussion to the early phase of the collision dominated by the sQGP. If the latter phase is close to strongly coupled  $N = 4$  SUSY matter at finite temperature, as two of us discussed recently [5], it is then useful to bring duality insights to bear on the bulk and kinetic properties of the sQGP.

## APPENDIX B: COMMENTS ON PLASMA BUBBLE

The formation of a metastable "plasma bubble" was recently discussed by Giddings [11] and its boundary and

slow evaporation of glueballs was discussed in Ref. [13]. In QCD, the idea that near the phase transition there can be a near-stable fireball has been discussed since the 1970's; see in particular Ref. [45]. Although at finite  $N$ , the pressure is always finite, the ratio to the energy density  $p/\epsilon$  has a minimum called “the softest point” [46]. If the system is produced at such conditions, the produced fireball will be especially long-lived; see the detailed hydrodynamical studies in [46]. Experimentally, there are indirect hints that fireball lifetime is, indeed, maximal around the collision energy  $\sqrt{s} \approx 6 \text{ GeV} * A$ , which unfortunately has not been studied in detail yet: there are proposals to run RHIC at such a low energy to verify that. The gravity dual of such slow evaporation is Hawking radiation in the (usual not extra) spatial direction. This phenomenon is known in QCD as a “long-lived fireball” It was first related with the MIT-type bag model, in which a meta stable zero-pressure state is possible. It was suggested as early as 1979 as a possible explanation for why the secondaries produced in pp collisions at ISR have thermal-looking spectra for transverse momenta without hydro-expansion [44]. Hydrodynamical transverse expansion was observed in heavy ion collisions in 1990's in AGS experiments in Brookhaven: its small magnitude was attributed to the so-called “softest point” of the equation of state, the minimum of  $p(e)/e$  right after the phase transition, which is close to the initial conditions in those collisions<sup>#5</sup>.

Aharony *et al.* [13] argued that since the critical pressure is  $p_c \approx N_c^0$  while the energy density on the plasma side of the transition is  $e \approx N_c^2$ , their ratio is  $1/N_c^2$ , vanishing in the limit of a large number of colors. Furthermore, they argued that a small but nonzero critical pressure can be further compensated by a surface tension  $\sigma$ , making stable plasma drops provided that the drops are small enough<sup>#6</sup>, with a size  $R < Rc \approx \sigma/p_c$ . For large  $N_c$  this can again be large because  $\sigma \approx N_c^2$ .

One important feature of the arguments presented in Ref. [13] is that the evaporation of glueballs from the surface of such plasma droplets leads to a reduction of the radius  $R$  in the ratio  $\sigma/p_c$ , so that inside the droplet the temperature should actually *increase*. This is indeed qualitatively similar to what happens with an evaporating black hole, emitting Hawking radiation.

While interesting, this however is a doubly tuned situation, which is rather different from what we expect at RHIC. In this case, there is a violent expansion and cooling of the fireball as we have attempted to describe in this paper. In our setting QCD matter (and its collisions) is placed as a test brane removed from the IR

limit of AdS. The expansion and cooling are both due to a warping of the metric induced by a departing black hole albeit far towards the IR.

## APPENDIX C: A FEW ESTIMATES

### 1. Collision Geometries

To simplify the initial geometry of the problem, imagine that colliding bodies may have infinite extensions in some directions, with the solution naturally independent of the corresponding variables. Let us call the number of “non-contracted” variables  $\tilde{d}$ .

The simplest geometry (i) would be a spherical collapse: One may imagine a spherical shell of matter collapsing into itself with an initial radial velocity  $v$  and Lorentz factor  $\gamma = 1/\sqrt{1-v^2}$ . The fireball That is produced in this case is expanding in a spherically symmetric way, producing a “Little Bang” like at RHIC, only in a much simpler spherical geometry<sup>#7</sup> The next geometry (ii) to consider is a collapse of a cylindrical shell, leaving one “non-contracted” variable,  $\tilde{d} = 1$ . The gravity dual to it should have a black hole with one less dimension. The geometry (iii) with  $\tilde{d} = 2$ , is a collision of two infinite 2D walls. This is close to what happens at RHIC, where the colliding Au nuclei are Lorentz contracted by a factor of a hundred into two thin pancakes. A variant of this is light-like wall-wall collisions that pass through each other, causing surface/string rearrangements in the minimal impact parameter region much like the parton-parton scattering approach originally suggested in Ref. [15] and [16].

The realistic <sup>#8</sup> case (iv) corresponding to RHIC is a collision of finite-size objects (although as large as practically possible). Due to relativistic boosts the nuclei get flatten in the collision direction  $x_3$ . Furthermore, for non-central collisions the overlap region is not axially symmetric but has an almond-like shape. Its gravity dual presumably would create a black hole with a horizon of some ellipsoidal shape, with different dimensions in all directions.

### 2. Fermi-Landau Model and the Entropy Formation

In QCD and other asymptotically free theories, we know that at small distances (close to the origin), the

<sup>#5</sup> Now there is a proposal under evaluation to run RHIC at very low collision energy to possibly produce such a fireball close to its critical state and study the “softest” region of a strongly coupled quark-gluon plasma experimentally.

<sup>#6</sup> This may explain why small systems, such as those produced in pp, do not show hydro-expansion while large ones produced in central PbPb collisions at similar energies do.

<sup>#7</sup> This has been considered by one of us many years ago [43] for e+e- collisions and prior to QCD. However, due to asymptotic freedom, this condition cannot be created experimentally: e+e- collisions, in fact, result in 2 jets, propagating from the collision points in random directions rather than a spherical expansion.

<sup>#8</sup> The gauge theory under consideration is still not QCD, but a strongly coupled  $\mathcal{N}=4$  SUSY YM.

interaction is weak. In the collision, the constituent partons literally fly through each other. Thus the issue of entropy formation at RHIC is complex and, as one may have suspected, not unanimous.

In contrast, in strongly coupled  $N = 4$  SUSY YM theory, there is no relation between the coupling and the scale. At strong coupling, one may think that the colliding matter is stopped and that most of the entropy is produced promptly at this stage. Thus, we use for this case the Fermi-Landau (FL) model [9,10] as a benchmark for further comparison.

The main assumption of the model FL is that matter can be stopped in a Lorentz-contracted size  $R = R_0/\gamma$ , where  $R_0$  and  $\gamma$  are the original size of the colliding objects and their Lorentz factor. The volume in which this is supposed to happen is

$$V \sim R_0^3/\gamma^{3-\tilde{d}}, \tag{C1}$$

where  $\tilde{d}$  is the number of “non-contracted” coordinates introduced in the preceding subsection. The first step of the argument is to evaluate the temperature at this stopped stage. The energy density is

$$\epsilon = E/V \sim \gamma^{4-\tilde{d}} \sim T^4 \tag{C2}$$

where the last equality is from the EoS of matter. Therefore the temperature grows with the collision energy as

$$T \sim \gamma^{1-\tilde{d}/4} \tag{C3}$$

The next step gives the amount of entropy produced:

$$S \approx T^3 V \approx \gamma^{\tilde{d}/4} \tag{C4}$$

One can see, that in the spherical collapse (i), there is no entropy growth because  $\tilde{d} = 0$ . The lesson from it is that only the cases with less trivial geometry provide some interesting predictions.

Despite the differences between the FL model and QCD, the entropy prediction for the wall-on-wall case (iii),  $S \sim \gamma^{1/2} \sim s^{1/4}$ , agrees with the observed multiplicity growth quite well. We will return to the discussion of this point later.

### 3. Black Hole Formation

For simplicity, we assume that the black-hole has been formed but that the flaking of closed strings is still taking place, and ask: under what conditions the flaking strings can be captured by the black-hole? what is the typical accumulated entropy? what is the typical time for this entropy formation? Most of the arguments in this subsection are heuristic.

From the AdS black hole metric we have

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_5^2, \quad \text{with} \tag{C5}$$

$$f = 1 - G_5 M/r^2 + (r/R)^2.$$

where we have ignored the distortions caused by the boundary brane on the black-hole. The size of the horizon of the black hole is

$$r_{bh} = R \left( \left( \frac{G_5 M}{R^2} + \frac{1}{4} \right)^{1/2} - \frac{1}{2} \right)^{1/2}. \tag{C6}$$

Hence  $r_{bh} = (R^2 G_5 M)^{1/4} := b$  for a large black hole, while  $r_{bh} = \sqrt{G_5 M}$  for a small black hole. #9 The temperature of the large black hole is given by  $T_{bh} = b/\pi R^2$ , while that of the small black hole is  $T_{bh} = R/2\pi b^2 \sim 1/\sqrt{G_5 M}$ . The large AdS black hole does not evaporate while small black holes can. However, the hawking temperature goes up as it evaporate while the fireball cools as it evolve. Therefore small black hole seems to be improper to describe the RHIC fireball. Therefore throughout this paper we identify the RHIC fireball with large AdS black hole.

We can express the mass and entropy in terms of Hawking’s temperature:

$$M = R^6 T^4 / G_5, \quad \frac{S}{V_3} = \frac{\pi^3 R^2 b^3}{2 G_{10}} \approx T^3. \tag{C7}$$

On the other hand, using  $G_5^{-1} = G_{10}^{-1} R^5 = M_p^8 R^5$ ,  $M_p = 1/l_s g_s^{1/4}$ , and  $R^4 = g_s N_c l_s^4$ , we can express Hawking’s temperature in term of the mass:

$$T = \frac{b}{\pi R^2} \approx \frac{1}{\pi \sqrt{N_c}} \left( \frac{M}{R^3} \right)^{1/4}. \tag{C8}$$

Since  $M \approx N_c^2$  (see below) then  $T \approx N_c^0$ . The time the entropy is reached typically corresponds to the falling time in AdS space

$$\tau = \frac{\pi}{2} R \tag{C9}$$

which is of order  $N_c^0$ . This is about the time it takes the final black hole to reach the *bottom* of AdS.

In a typical RHIC experiment, hundreds of nucleons or thousands of quarks are involved as shown above. As the interaction is almost simultaneous, thousands of interaction vertices are involved and a shower of massive closed strings are created and fall into the center of the AdS space. Let  $N = \pi R_N^2 Q_s^2$  be the number of such collisions with  $Q_s^2$  their transverse density and  $\pi R_N^2$  the transverse nucleon size. In field theory charged quantas moving with rapidity  $Y$  are surrounded by extra quanta distributed at smaller rapidities  $dy = dx/x$ . In QED the Weiszacker-Williams approximation yields a flat distribution of these quantas versus  $y$ , *i.e.*  $dN/dy$  constant. In QCD  $dN/dy$  is not constant and behaves approximately as  $e^{\alpha(t)(Y-y)}$ . HERA data suggests  $\alpha \approx 1/4$  at  $t \approx -1 \text{ GeV}^2$ . In weak-coupling the BFKL approximation gives  $\alpha_{BFKL}(0) = (4\alpha N/\pi) \ln 2$ , while at strong

#9 large black hole means  $G_5 M/R^2 \gg 1$  with  $f \sim (r/R)^2(1 - b^4/r^4)$ , and small black hole means  $G_5 M/R^2 \ll 1$  with  $f \sim 1 - G_5 M/r^2$ .

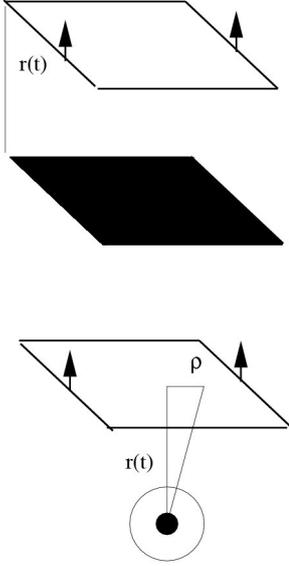


Fig. 7. Sketches of the brane motion for the Big Bang (a) and Little bang (b) geometries. (a) refers to a brane moving away from large “black brane” with a time-dependent distance in 5-th dimension  $r(t)$ . (b) refers to an asymptotically flat brane with the brane to black hole distance  $r(t)$ , while  $\rho(t)$  is the effective spherical size of the fireball on the brane. The Howking radiation has homogeneous time-dependent temperature in (a), while it depends on  $\rho$  in case (b).

coupling arguments based on AdS/CFT duality yield  $\alpha_{AdS}(t) \approx 7/96 + 0.23t$  [16]. In our case, we will use a transverse parton density  $dN/dydx_{\perp} = Q_s^2(y)$  with  $Q_s$  being, in general,  $y$ -dependent.

Since the 5th AdS coordinate  $r$  is orthogonal to the boundary collision axis, only momenta of closed strings with typically  $Q_s$  are relevant. Then the typical total energy of the closed strings is  $M \approx \int dy dN/dy Q_s$ , with all strings assumed to be created instantaneously at the impact. The energy of the closed string must be identified as the radial coordinate in the AdS space. The strings flake towards the center of the curved AdS space under gravity and arrive at the central region simultaneously. The average energy per string is  $\epsilon \approx Q_s$ . The total en-

ergy is therefore of order  $NQ_s$ . We note that  $N \approx N_c^2$  and  $Q_s \approx N_c^0$ , so that the total energy is of order  $N_c^2$ . A black-hole forms when the horizon radius is bigger than the size of the closed string. For the large black hole, the horizon distance is  $r_{bh} = (R^2 G_5 M)^{1/4}$ , where  $G_5$  is the 5 dimensional Newton’s constant. Hence the black hole formation condition is

$$Q_s^{-1} \leq (R^2 G_5 M)^{1/4}. \quad (C10)$$

In terms of  $N(s)$ , the number of pair collisions at the boundary, Eq. (C10) reads

$$N \geq R^{-2} G_5 Q_s^{-5}. \quad (C11)$$

We recall that  $M \approx Q_s^3$  and  $N \approx Q_s^2 \approx s^{\alpha}$ . The entropy generated at the surface by the RHIC collision depends on the collision energy as follows:

$$S \approx T^3 V_3 \approx s^{9\alpha/8-1/2} \quad (C12)$$

for an initial temperature  $T \approx s^{9\alpha/8}$ .

#### APPENDIX D: A CORRECTION TO THE BIG BANG PICTURE

The situation is schematically illustrated in Fig. 7 (b). The temperature at the center of the fire ball is given by the warping factor as determined above using the distance  $r$  from the black hole center to the fire ball center. The temperature at all other points in the moving brane is warped further since the effective distance now is  $\sqrt{r_{\perp}(t)^2 + \rho^2}$ , and vanishes asymptotically. While the precise determination of the imbedding of the moving brane is very involved, its shape in the region far from the black hole can be readily discussed approximately by neglecting the bending effect. More precisely, from the metric

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_5^2, \quad \text{with} \\ f = 1 - G_5 M/r^2 + (r/R)^2, \quad (D1)$$

the temperature is approximately given by

$$T(t) = T_0 \sqrt{f(r_0)} / \sqrt{1 - G_5 M / (r_{\perp}(t)^2 + \rho^2) + (r_{\perp}(t)^2 + \rho^2) / R^2}, \quad (D2)$$

where  $T_0$  is the black hole temperature at a reference distance say  $r = r_0$ . Now, the temperature depends both on the spatial size  $\rho$  of the fire ball on the boundary, and the distance to the black hole  $r_{\perp}(t)$ . This is the temperature profile which has a peak at the fireball center,  $\rho = 0$ , and decreases as  $\rho$  increases. See Fig. 8. The upshot of this analysis is that the presence of a distant

black hole in bulk produces small additional forces on moving matter in the moving brane. Even without considering the bending effect of the brane, the presence of black hole is detected through the metric. For instance, hydrodynamical flow of matter on the brane is now described by  $T_{\mu;\nu}^{\nu} = 0$ . The source of the Christoffels  $\Gamma$  are two fold: one is the expansion induced by the motion

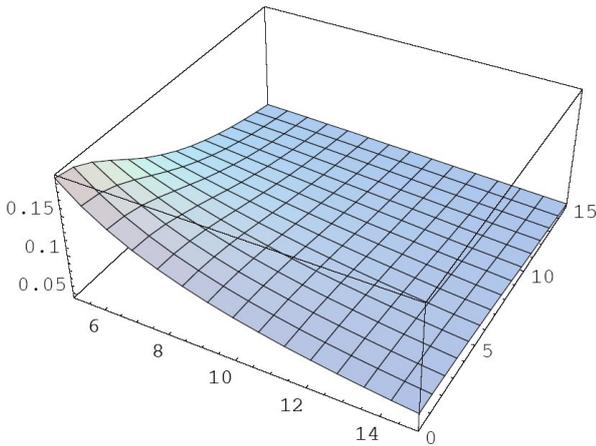


Fig. 8. Plot of the temperature for  $0 \leq \rho \leq 15$  as the brane position  $r_{\perp}(t)$  moves from 5 to 15. The bending of the brane is ignored assuming that the brane is far from the black hole. We have used  $G_5 M = 1$ ,  $R = 1$ ,  $T_0 \sqrt{f(r_0)} = 1$ . Notice that the warping factor ( $\sim r^2$ ), which will overcompensate the apparent shrinking in the fireball size, is not taken into account in the plot.

of the brane inside a warped background, the other is due to the presence of the black hole. These modifications to ideal hydrodynamics are not small even at late stages as far as the strong character of the interaction is sustained. However, these effects are small far from the fireball center at all times. This analysis will provide yet another route to non-ideal hydrodynamics for  $\mathcal{N} = 4$  SYM theory at strong coupling and in particular, to the calculation of the transport coefficients. This will be reported elsewhere.

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